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Calculus students with different curricular histories: a look at past mathematics experiences, learning preferences, and understanding of various representations of derivatives

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Calculus students with different curricular histories: A look at past mathematics experiences, learning preferences, and understanding of various representations of derivatives

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Signatures have been redacted for privacy

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INTRODUCTION

Reform efforts in United States mathematics education currently are focused on the teaching of mathematics in elementary and secondary schools (National Council of Teachers of Mathematics, 1989, 1991, 2000). Some high schools have switched their curriculum materials to fit these reforms, some have not, and some school districts offer the choice of learning from either reform mathematics curriculum materials or a Traditional curriculum. Mathematicians are recognizing that calculus is immensely important to mathematics and are becoming involved in incorporating concepts in calculus that will aid in the understanding of calculus in earlier grade levels (Young, 1986). This thesis focuses on students' understanding of the different representations of derivatives from their different past curriculum material experiences, with the intent of providing a clear understanding of students' current experiences based on their prior experiences. Additionally, this thesis examines the confidence of students during interviews when solving problems which draw on different representations of derivatives.

This thesis examines students who attend a school in which a choice of curricula are being offered. The overarching goal of this study is to examine the nature of the similarities and differences between students who are currently taking calculus, and who come from two different past curricula. This study also investigates how the students solve problems with different representations of derivatives. Some students took a "Traditional" sequence of mathematics courses. (Throughout this thesis, I use the word "Traditional" to refer to the sequence of courses the students took that include Algebra, Geometry, Algebra II, Pre-calculus. While I recognize that the teaching in these courses may not have been "traditional", the textbooks that were used were written prior to the publication of The

National Council of Teachers of Mathematics (NCTM) *Standards* documents. Also, the school district itself called this sequence the "Traditional" sequence because the courses separated the content strands into separate courses.) Other students took an "Integrated" sequence. (Similarly, I use the word "Integrated" to refer to the courses which used the *Core Plus* (Hirsch et al., 2001) curriculum. These courses integrate topics from algebra, geometry, pre-calculus, and statistics. Again, I use this term because this is what the school district calls it; I am not claiming that the teachers of these courses taught in a reform-oriented way, but the curriculum materials were written in the spirit of the NCTM documents.) More specifically, this thesis explores students' confidence and performance ability when solving problems involving derivatives. Because the students' past curriculum materials experience were different, this study seeks to understand how these past experiences contribute to their understanding and confidence in working with the concept of derivatives.

This chapter highlights background information regarding the study, including its purpose, the change in teaching and curriculum materials that have occurred over the past ten years, and the possible influence of curriculum materials on teaching. This chapter also includes the research questions driving this thesis.

Purpose of Study

Students who enroll in a Traditional sequence are likely to experience different teaching methods and curricula than those enrolled in an Integrated sequence. When students enroll in a Traditional sequence or in an Integrated sequence, it is likely that they experience different teaching and curricula. However, I found no research on what happens when these two types of students come together to take a calculus course after these different curricular experiences. Additionally, there are even fewer studies that examine the students' perspectives (Lubienski, 2004) of the similarities and differences between the different curricular experiences. If there are no differences among students in the same class who have taken an Integrated sequence and students who have taken a Traditional sequence, one might question whether the objectives of reform mathematics are being met. It may be that the textbooks are not being implemented in the way they were intended. Or, it may be a problem with the methods of inquiry that were used to decide what is different (e.g., standardized tests may show that the students basically score the same number). An interview with each student, however, may detect differences that do not appear on standardized tests in terms of how well they can explain and articulate their thinking.

Reforming Mathematics Teaching and Learning

Change in Teaching.

In recent years, educators in mathematics have expressed that a shift in teaching is needed if current reform documents (National Council of Teachers of Mathematics, 1989, 1991, 2000) are to be realized in the classroom (Nelson, 1997). The shift involves moving from Traditional to what many refer to as "reform oriented" forms of teaching mathematics. Smith III (1996) describes Traditional mathematics teachers as those who "provide clear, step-by-step demonstrations of each procedure, restate steps in response to student questions, provide adequate opportunities for students to practice the procedures, and offer specific corrective support when necessary" (p. 390). In contrast, reform oriented teaching of mathematics can be characterized as the teacher being a facilitator, posing questions that engage and challenge each student's thinking and asking students to justify their ideas. The instruction stresses group work and learning from peers (Smith & Burdell, 2001).

While some school educators will continue to favor the Traditional way of teaching mathematics, others strongly view mathematics as an opportunity to help children become good problem solvers who can understand relationships through explorations, reasoning, and exchanging ideas with others, which are essential skills in real life.

Change in Curricula.

This section looks at some of the changes in curriculum materials developed with funding from the National Science Foundation (NSF) in the 1990s. These materials were developed to embody the ideas put forth by the NCTM documents. These differences are described so that the reader will understand some of the differences students may have experienced in their "curriculum material history." Once some of the broader differences are described, the study then looks more closely at *Core-Plus Mathematics Project* (CPMP) and The University of Chicago School Mathematics Project (UCSMP), which are the curriculum materials the students in this study have taken prior to their calculus course. The UCSMP textbooks maintain an emphasis on separating Algebra, Geometry, Algebra II, and Pre-calculus. For this reason, the school district in which this study took place refers to the curriculum as the "Traditional sequence." The CPMP textbooks, on the other hand, are based on NSF funded curricula and have integrated these and other topics throughout the four years of the curricula. For this reason, the school district in which this study took place refers to the curriculum as the "Integrated sequence."

In the early 1990's, NSF released a solicitation for curriculum development. They sought to improve the quality of learning and teaching in mathematics classrooms by incorporating a curriculum that has an emphasis on problem solving. The *Standards* documents articulate a vision of quality mathematics education in which students learn important mathematical concepts and processes with understanding. Thus, any curriculum consistent with the *Standards* must contain materials that present mathematical concepts in a coherent, well-articulated fashion through problem solving contexts.

Over the years, the curricula in mathematics have changed. For example, Star, Herbel-Eisenmann, and Smith (2000) identify some differences between older and newer algebra curricula by examining some of the dimensions of one of the middle school reform curriculum materials, the *Connected Math Project* (CMP) and more Traditional algebra curriculum materials. Some typical problems in algebra curricula include verbs such as "factor", "solve", "multiply" and emphasize mainly symbolic expressions. In contrast, CMP's problems included verbal statements with graphs, tables, or symbolic expressions with requests to "describe", "explain", "predict", etc. Some of the verbs used in the two curricula could represent the cognitive tasks of Bloom's Taxonomy. The Traditional curriculum materials' verbs represent the Application Level, whereas the CMP curricula represent Analysis, Synthesis and Evaluation; this article illustrated that the newer curricula focus often on a higher order of thinking. The solution methods required of students were also different between the two types of curricula. The Traditional curricula require students to complete the correct steps in symbolic procedures in the correct order, while the CMP curricula give the students the opportunity to relate verbal statements to tables, graphs, or equations, to compute different representations, and to interpret the results verbally.

The CPMP curriculum is designed to make mathematics meaningful and accessible to all students (Schoen & Hirsch, 2003). CPMP is an Integrated mathematics curriculum which emphasizes the theme of using mathematics to make sense of the world around us. The CPMP curriculum has many problems that include graphical representations, and most problems require students to reason with multiple representations and answer questions with convincing arguments (Huntley et al., 2000).

The CPMP curriculum has "Investigations" sections, which are problems that actively engage the students in investigating and making sense of problem situations. Investigation sections are followed by a set of homework-like tasks that are referred to as "Modeling with Organizing, Reflecting on, and Extending" (MORE) mathematics understanding. "Modeling" tasks relate to or provide new contexts to which students can apply the ideas and methods that they have developed in the lesson. "Organizing" tasks offer opportunities for integrating the formal mathematics underlying the mathematical models developed in the lesson and for making connections with other strands. "Reflecting" tasks encourage thinking about thinking itself, about mathematical meaning, and about processes. These reflecting tasks promote self-monitoring and evaluation of understanding. "Extending" tasks permit further, deeper, or more formal study of the topics under investigation (Hirsch et al., 2001). All of these types of problems are aligned with the *Standards* put forth by NCTM.

In the UCSMP curriculum materials, the application of mathematics is incorporated into every chapter to motivate the content for the students and help develop abstract concepts. The chapters first begin with a section describing the content, which is followed by examples. The textbook has homework sections titled, "Questions Covering the Reading" and "Notes on Reading" that are designed to help students read efficiently (Senk, 2003). These two sections are designed to check the students' understanding of the reading presented before the mathematics topics discussed. Depending on the content, not all of the questions are focused on mathematical ideas; for example, some questions are related to a story that is told at the beginning of the chapter. Also, "Questions Applying the

Mathematics" is designed to encourage students to extend the concepts presented in the lesson, while also providing some practice problem on the content presented earlier in the chapter (Senk, 2003). Many of these questions are word problems. The "Review" section gives the students practice on ideas that have been studied earlier. This is where the words "solve", "evaluate an expression", etc. appear. Finally, there is an "Exploration" section that allows the students to explore various areas in the same mathematics content. Most of the verbs used in this curriculum are those used in the Traditional algebra curriculum described by Star, Herbel-Eisenmann, and Smith in the article discussed earlier.

Influence of Curriculum on Teaching

Changing teachers' practice is difficult (Hiebert, 1999; Richardson, 1990). Developers of the *Standards* based curriculum materials have a goal of providing teachers with materials that encourage them to align their teaching practices with the NCTM *Standards*. For example, the materials may suggest to the teacher that he/she has students work in small groups to discuss mathematics, or request that students explain their reasoning and come up with multiple approaches to solving a problem. In *Standards* based curricula, lessons are often introduced by presenting students with an unfamiliar problem rather than a worked example (Goldsmith et al., 1998). Although materials alone cannot change teacher practice, they can provide scaffolding for teachers trying to create a classroom environment different from the norm (Hamilton et al., 2001).

While it is evident that curriculum materials can impact the teaching that occurs in the classroom (Lloyd & Wilson, 1998), this study makes no claims that this is the case. This study focuses on students who have experienced the different curriculum materials described and investigates how these past curricular experiences have possibly influenced their calculus learning experiences.

Research Questions

The overarching question examined is: What are the similarities and differences between students from a Traditional curricular experience and students from an Integrated curriculum sequence who are now taking calculus together?

More specifically, this study addresses the following questions:

a. How do the students describe their past mathematics experiences with respect to how it impacts their calculus experience?
b. How do the students perform on a set of problems involving various representations of derivatives?
c. How confident are the students when solving a set of problems involving various representations of derivatives?

Now that the purpose of this study and the research questions have been discussed, some history about mathematics reform, occurrences of reform in mathematics teaching, and learning and its influences, the literature closely related to this study will be examined.

LITERATURE REVIEW

This study describes the similarities and differences of Traditional and Integrated students' past experiences. In doing so, it focuses on whether and how those different curricular experiences a) influence their current experiences in calculus, b) impact their confidence when solving problems involving derivatives and c) shape their dispositions after solving problems involving derivatives with different representations. To help address these issues, this study examines the literature that discusses the impact of reform-based and Traditional curriculum materials on students. First, the impact of the curricula is discussed more broadly and then the study delves more specifically into what has been written about CPMP and UCSMP. In this section, the study provides some history of the reform movement in mathematics, and then it provides a look at the literature related to the curriculum materials from which the students come and at research about defining students' confidence and their ability to solve different representations of derivatives. Lastly, the study provides a definition of students' confidence and of derivatives and the different representations that exist, using the Rule of Four.

Historical Context

In 1989, the NCTM published the *Curriculum and Evaluation Standards* in an attempt to improve school mathematics instruction. The 1989 document specified fourteen *Standards* for grades nine through twelve. Those *Standards* included four process *Standards*: *a)* mathematics as problem solving; *b)* mathematics as communication; *c)* mathematics as communication; and *d*) mathematical connections. They also included ten content *Standards*: *a)* algebra; *b)* functions; *c*) geometry from a synthetic perspective; *d)* geometry from an algebraic perspective; e) trigonometry; *f)* statistics; *g)* probability; h) discrete mathematics; *i)* conceptual underpinnings of calculus; and lastly, *j*) mathematical structures. These *Standards* established a core curricular framework that the NCTM believed would satisfy the needs of all students and help them succeed in mathematics after secondary schooling.

In 1989, the organization also produced a summary of changes in content of topics to be emphasized in mathematics. Within the above listed content areas, certain topics were to receive increased or decreased attention in the classroom. Some topics that the authors advocated to receive increased attention included: 1) using real-world problems and computer-based methods in algebra, 2) integration of content topics within mathematics across all grade levels, 3) deductive reasoning, 4) real-world applications in geometry, 5) the use of scientific calculators, 6) realistic applications and graphing utilities in trigonometry, 7) making connections between functions and real-world problems, 8) statistics, 9) probability, and 10) discrete mathematics. Instructional practices also were to receive increased attention. Some of the practices teachers were asked to increase included: 1) active student involvement, 2) problem solving, 3) effective questioning techniques, and 4) the use of calculators and computers as tools. Instructional practices receiving decreased attention included: 1) teacher and text as exclusive sources of knowledge, 2) rote memorization of facts, 3) extended periods of individual seatwork, and 4) paper and pencil manipulative activities (NCTM, 1989). To support these increases and decreases, new textbooks were published.

Impact on Students

This section focuses on studies of confidence and students' achievement and the impact of curriculum materials on students in the mathematics classroom. Specifically, it focuses on studies that examine students' achievement with a *Standards*-based curriculum, more broadly, and then focus on CPMP curricula and UCSMP curricula. Lastly, it provides a definition for students' confidence and discusses studies related to assessing students' confidence.

Student achievement in Standards-based curriculum

Usiskin et al. (2002) calls for changes to occur in the secondary classroom; this includes upgrading students' performance to include more problem-solving instead of rote memorization (Thompson & Senk 2003). Boaler (1998) found that students exposed to instruction that emphasized some of the ideas put forth in the *Standards* outperformed students who were taught using procedures and memorization in a range of assessments and situations. Stein and Lane (1996) conducted a study that showed there was an increase in student learning when taught using reform-based instruction aligned with the *Standards*. These studies provide evidence that the change in teaching that is taking place in some schools should be occurring and is effective in enhancing student learning.

Boaler (1998) found that students exposed to instruction which emphasized flexible thinking and application of mathematics outperformed students whose instruction emphasized procedures and memorization in Algebra. She presented a short written test for the students to complete, which showed that the students were equally capable of completing the tasks. However, where the students had to apply the knowledge they learned, the students in the reform based school performed better. The students from the Traditional school agreed that they could not interpret the demands of the different questions and they could not apply the procedures they had learned to the questions Boaler asked.

A study by Stein and Lane (1996) revealed a relationship between gains in student learning and *Standards*-based instruction. In particular, they examined the use of mathematics tasks with multiple solutions strategies, multiple representations, and explanations. Thompson (2001) found that students in *Standards*-based curriculum outperformed students in Traditional curriculum on multi-step problems and problems involving applications or graphical representations. From these studies, there may be a promise of reform-based instruction to improve students understanding in mathematics.

Smith et al. (2000) conducted a study that examined transitions of students from a *Standards* based curriculum to a Traditional curriculum and vice versa. They studied students' perspectives about moving from a *Standards* based curriculum to a Traditional curriculum used in a high school and a college. They also studied students moving from a Traditional curriculum to a *Standards* based curriculum in a high school and a college. In relation to their prior experiences, the students noticed differences between the curricula. Many of the students reported that they had more "story" or "word" problems in their curriculum. The students, who preferred the Traditional curriculum, complained about the "story" problems being harder because the problems tracked their understanding of the content. They also reported that students who came from and went to a *Standards* based curriculum spent more time explaining their answers to the problems than they did in their Traditional curriculum counterparts.

All of these studies examined *Standards* based curricula and its effects on student achievement.

Students' Understandings in CPMP.

Positive results in student achievement have been observed for several mathematics curriculum materials that focus on problem solving (Cobb et al. 1992; Carpenter et al. 1998); problem solving is a characteristic of *Standards* based curricula. Huntley et al (2000) conducted a study on the effects of the *Standards* based CPMP and how they compared to a more Traditional curriculum. Students who were matched on measures of 8^{th} grade achievement in CPMP Integrated III and in more Traditional Algebra II classes were administered a researcher-developed test of their algebraic understanding, problem solving and procedural skill at the end of the school year. CPMP students scored significantly better on the sub-tests of understanding and problem solving, and Algebra II students scored significantly better on the sub-test of paper-and-pencil procedures[1].

Over the past years, research has been done by the authors of CPMP. There may be a conflict of interest, so some of these issues need to be studied by outside people who do not have anything to gain from positive results. Schoen and Hirsch (2003a) have conducted or have cooperated in the conduct of various studies in mathematics achievement in CPMP classrooms. When comparing students' results from an *Algebra End-of-Course Examination* of CPMP curriculum to students from the Traditional curriculum, they found that in nearly every comparison, CPMP students almost always outperformed comparison students on conceptual understanding, interpretations of mathematical representations and problem solving in applied context. However, they sometimes did not do as well on measures of algebraic manipulation skills. Their performance is also relatively strong in content areas like probability and statistics.

More specifically related to calculus, Schoen and Hirsch (2003a) examined the level of preparation that students were given after using each of the two sets of curricula. They examine each of the curriculum materials and argue that Integrated IV covers more material

[1] These materials are being revised and the authors are taking these kinds of finding into consideration in the revision

for preparation for success in calculus, than Pre-calculus, a course taken by students in the Traditional curriculum. They also administered a calculus-readiness exam to students. All of the students in the Traditional curriculum scored higher on symbol manipulation procedures that are commonly emphasized in the Traditional curriculum. CPMP students performed at higher levels on problems involving using conceptual knowledge and problems involving application.

Schoen and Hirsch (2003b) conducted another study, in a different high school from their previous study, where they administered a pre- and post-test to students using Traditional curriculum materials and CPMP curriculum. They found the same results as the study discussed in the previous paragraph. In addition, they also showed evidence that students taking the CPMP curriculum materials were just as prepared for the SAT and ACT college entrance examinations than students in the more Traditional curriculum. This study also verified that CPMP students are somewhat weaker in decontextualized, paper-and-pencil symbolic manipulation.

A study by Smith and Burdell (2001) looked at the transitions students experienced who had a Traditional curriculum in junior high and then moved into a reform curriculum in high school. The students noticed mathematical differences between the two sets of curricula. The students stated that the contextual problems were more difficult. They also were asked to explain their thinking more and were required to work in groups more than they had in their past classes. The students described the work in the Traditional classroom as requiring them to memorize the information. This study also provides evidence that students can be more adaptable to educational changes that occur in school such as working in groups more often and being required to explain their thinking more thoroughly. They also found that the students noticed the differences between the two sets of curriculum materials through the nature of the problems and the thinking required to do the problems.

Students' Understanding in UCSMP.

Hirschhorn (1993) conducted a longitudinal study to examine the achievement of students that used the UCSMP curriculum. Based on two post-tests and a survey, results showed that students using the UCSMP curriculum performed better on standardized tests than students who did not use the curriculum. USCMP students also outperformed the other students on problems of applications of mathematics.

Thompson and Senk (2001) compared UCSMP students to students that did not use the UCSMP curriculum on a multiple-choice test on Advanced Algebra. The UCSMP students outperformed the other students on a pre- and post-test.

The authors of UCSMP conducted various studies on students who use the curriculum. They found that UCSMP students perform better than students not using the curriculum materials when solving algebraic problems presented in meaningful contexts. The authors presented data on the UCSMP students' ability to interpret solutions and found that they can interpret a solution after they find the answer to the problem.

A study in Kilpatrick (2002), conducted by the UCSMP, used a test that was developed by them to compare UCSMP students to non-UCSMP students in Algebra to test problem solving and understanding. UCSMP students outperformed the non-UCSMP students on topics such as applications leading to quadratic equations and comparing quadratic functions, etc. They concluded that the UCSMP students maintain their hold on Traditional advanced algebra skills while enriching their advanced algebra background in the application of algebra and in problem-solving and understanding of mathematics. No studies conducted on the UCSMP Pre-calculus book could be found for the purpose of this study.

A study conducted by McConnell (1990) looked at the performance of UCSMP sophomores on the Preliminary Scholastic Aptitude Test (PSAT), given at Glenbrook South high school. The math section of the PSAT, which she studied, is said to be a preparation for the Scholastic Aptitude Test (SAT). This is a multiple choice exam that measures basic knowledge of Number and Operations, Algebra and Functions and Geometry and Measurement. She found that the students performed well on all of the sections compared to the national average of the students who took the exam.

Students' Confidence

While all affective variables are very important to students' learning, this study concentrates on the confidence that students articulate in interviews. Affective variables can influence the learning environment in a classroom in substantial ways. Affective variables can play an important role in students' decisions about how much mathematics they will need in the future and how they approach the mathematical content they do study (Reyes 1984). Current research on confidence is very scarce because recent studies of confidence continue to produce patterns of results established in earlier research efforts (McLeod 1992).

Reyes has studied confidence for many years and is well respected for studying affective variables; therefore, her definition of confidence is the one that is used in this paper. Reyes (1984) defines confidence in learning mathematics as "how certain a person is about her ability to learn and do problems on topics in mathematics, perform well in a mathematics class, and do well on mathematics tests" (p. 560). In other words, a person, based on his/her ability to do problems and perform in the classroom, determines whether they are confident in mathematics. Confidence is important to study because it can determine what type of problem a student likes or wants to solve. If a student is not confident in solving certain types of problems, they may not be persistent in finding the correct answer.

Reyes (1984) found that confident students tend to learn more, feel better about themselves, and be more interested in pursuing mathematical ideas than students who lack confidence. Confidence in students learning mathematics is important to study because, as Reyes (1984) says, confidence can help a student determine whether he/she will stay interested in studying mathematics. In order to understand the confidence of a student, it is helpful to understand the construct of self-concept.

Self-concept can be defined as an individual's perception of self (Shavelson, Hubner & Stanton 1976). For this study, I use the term "self-concept" to refer to an individual's perception of their achievement and ability in school. Several studies have been conducted to help determine the relationship between a student's self-concept and her mathematical achievements (Aiken 1976; Rubin 1978; Zeitz 1976). A students' confidence can be considered one aspect of self-concept because confidence, as defined in this study, is determined by how someone perceives their own ability.

Studies show that affective variables such as confidence play a central role in mathematics learning (Schoenfeld 1989; McLeod 1992). A study conducted by Prawat and Anderson (1994) examined the affective experiences, primarily confidence, of students engaged in mathematics tasks during class. Through interviews, they found that the students who performed well on the tasks were confident.

A study conducted by Dowling (1978) uses a survey instrument called Dowling's Mathematics Confidence Scale to measure the confidence in doing mathematics of females attending college. This scale presents mathematics tasks and one value is scored based on the confidence of the students' ability to solve the problem, another score is based on what the student says, and a third score is based on the whether the student can answer the mathematics task and whether their answer was correct. Dowling found a significant correlation between students' confidence and mathematics ability on tasks. The students were confident on problems that they had answered correctly. Confidence was measured based on whether the students verbally said they were confident.

There is a well established relationship between confidence and achievement in mathematics education literature. In this set of studies, the researchers' defined confidence in terms of persistence and the level of effort the participants applied towards tasks (Bar-Tal 1978; Dweck & Goetz 1978). One can assume from the findings in these studies that students with high confidence tend to achieve at a higher level. Both studies were quantitative; persistence and the level of effort was measured based on the participants' behavior while working on tasks given to them. For example, points were given to students based on the persistence and effort that was put forth. The limitations of the studies were that the researcher could have been subjective when determining the number of points given to a student, because many of the points were measured on what the students said in the interview conducted. Effort and persistence were measured with respect to how long it took them to work on the problem and if they verbally stated how much effort they put forth as well as whether they were persistent in trying to solve the problem.

Derivatives

Brief History of Derivatives.

Mathematicians all over the world have contributed to the developments of calculus, but Isaac Newton and Gottfried Wilhelm Leibniz are two discoverers that are most recognized. Currently, credit for discovering calculus is being given to both men; this was not always the case, however. In the past, people have tried to figure out which one deserved the credit. There was an assumption that Leibniz may have collected some of his ideas from two of Newton's manuscripts, and that that is what sparked his understanding of calculus. Many believed that Leibniz used Newton's ideas, which he got from unpublished letters they had written to each other. They believed that Leibniz created a new notation and then published it as his own, which would obviously represent plagiarism. Even though Leibniz and Newton were writing to each other, it was found that Leibniz had already written down some ideas of his own about calculus. As Leibniz and Newton exchanged letters, the ideas exchanged between the two could have helped Leibniz develop his own ideas. (Boyer, 1968).

Newton explored the use of applications of calculus further than Leibniz, while Leibniz concentrated more on discovering formulas that are used in calculus (Struik, 1948). Newton acknowledged the help he got from other mathematicians in a letter he wrote to Robert Hooke dated February 5, 1876. In it,, he stated, "If I have seen further, it is by standing on the shoulders of giants" (Merton, 1965, p.25). "Newton's *fluxional calculus* and Leibniz's *differential calculus* had quite different goals. Newton's ideas consisted of flowing quantities with a velocity, or rate of change over an infinitely small time" (Katz, 1993, p. 89). Newton studied more of the physical characteristics of calculus in relation to time. Leibniz, however, was more concerned with, "inventing the expressions of the derivative to indicate the differences of two infinitely close values of *x* and *y* respectively. Leibniz was also responsible for deriving many of the differentiation rules" (Katz, 1993, p. 98) used in calculus. While it is still unclear of who discovered calculus first, both Newton and Leibniz made great contributions to the advancement of mathematics, and both deserve credit for that.

Defining Derivatives.

A function is defined as "a mathematical relation such that each element of one set is associated with at least one element of another set" (Stewart, 1997, p. 101). To define derivatives, knowledge of slope can be helpful (Hughes-Hallett, D., Gleason, A., et al. 1994). Finding the derivative, in this definition, is like answering the question, "How do we measure speed?" (Hughes-Hallett, D., Gleason, A., et al. 1994, p. 94). In other words, one tries to calculate the speed of an object in a parabolic motion. The graph below further illustrates the definition of a derivative. For example, an object is being thrown in the air. Figure 2.1 displays the height of the object plotted against time. The *y* coordinates represents the height of the object. The *x* coordinates represents the time.

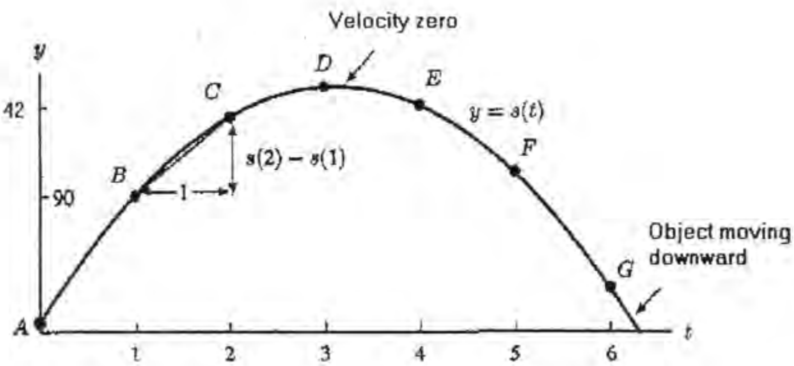

Figure 2.1: The height, *y*, of the object at time, *t*
Note. Source: (Hughes-Hallett, D., Gleason, A., et al. 1994, p. 97).

First, the average velocity of the object on this graph over the interval, $1 \le t \le 2$ needs to be identified.

To find this, the following equation is useful,

$$\text{Average velocity} = \frac{\text{Height Gained}}{\text{Time Elasped}} = \frac{s(2)-s(1)}{2-1} = \frac{142-90}{1} = 52 \text{ ft/sec.}$$ Hughes-Hallett, D.,

Gleason, A., et al. (1994) described the process this way,

> $s(2) - s(1)$ is the change in height over the interval, or the height gained. The 1 in the denominator is the time elapsed and is marked in the graph above. The average velocity is the same as finding the slope between the two points (1,90) and (2, 142) (p. 97).

From figure 2.1, it can be concluded that the average velocity is the slope of the line joining BC.

Imagine taking a graph and "zooming in" to get a close- up view around the coordinates (1, 90) and (2, 142). The more you zoom in, the more the curve looks like a straight line.

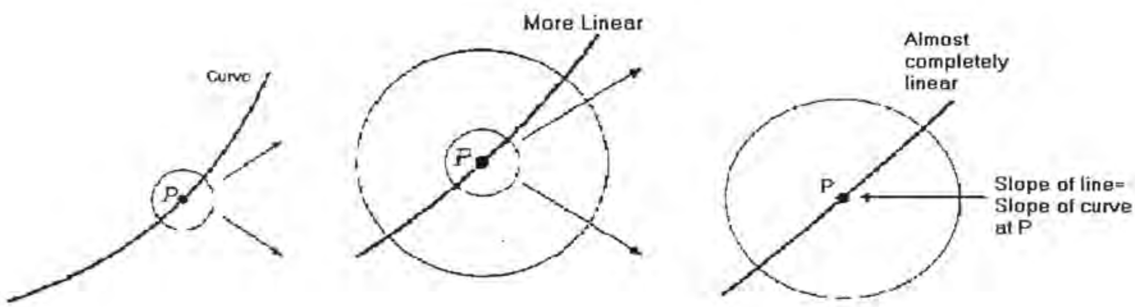

Figure 2.2: Finding the slope of the curve at the point by "zooming in
Note. Source: (Hughes-Hallett, D., Gleason, A., et al. 1994, p. 98).

You can call this the slope of the curve at the point. This is referring to the slope of points B and C in Figure 2.1. Hughes-Hallett, D., Gleason, A., et al. (1994) described this as:

> The slope of the magnified line is the instantaneous velocity, which is an object at time *t* given by the limit of the average velocity over the interval as the interval shrinks around *t*. Looking at Figure 2.1, at points A and B there is a positive slope. At point E, the object reaches its peak, the slope of the curve is zero; at point E, the curve has a small negative slope indicating its decent to the ground. At point D, the velocity is zero, at that point the object changes direction- from moving upward to moving downward- the object's instantaneous velocity is zero, meaning at that one moment of time, the velocity is zero (p. 98).

Both figures help show the slope of two points of an object in motion in small intervals.

Thus far, this study has defined that the velocity of an object at an instant *t* is to look at the average velocity over smaller and smaller intervals containing *t*. As before, let *f* be this name for the function of time that gives the height *y* of the object at time *t*, so $y = f(t)$. Using more general terms to form the definition of derivatives, refer to Figure 2.2 below.

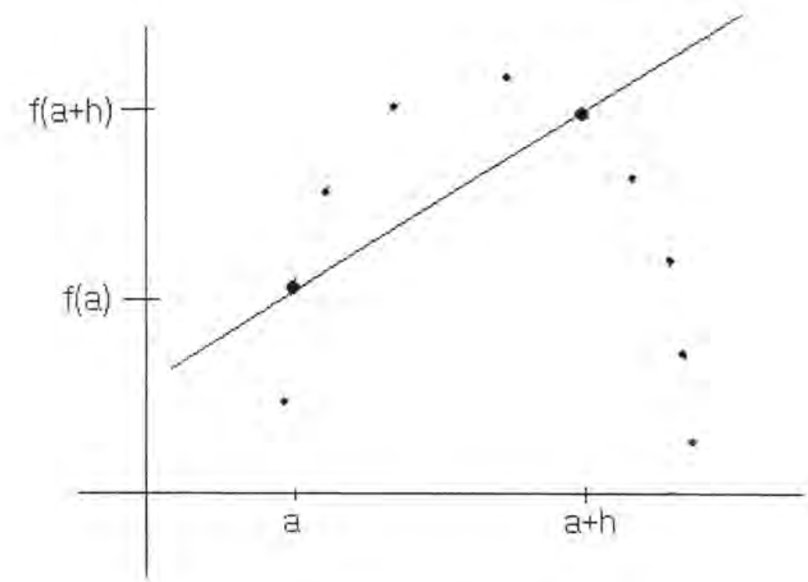

Figure 2.3: Finding the slope between two points

Using the average velocity formula, the following equation is derived:

Average velocity = $\frac{f(a + h) - f(a)}{h}$. In this graph the interval is $a \le t \le a + h$. The h in the denominator is derived from the distance between a and $a+h$. Instantaneous velocity was previously defined as the number that the average velocities approach as the intervals decrease in size, as h becomes smaller. So, the limit as h approaches 0, of $\frac{f(a + h) - f(a)}{h}$.

Thus, this gives the definition of derivative as $\lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$ (Stewart, 1997).

Representations of Derivatives

Many calculus reform texts aligned with the NCTM *Standards* have emphasized the use of multiple representations as a way to develop student understanding (Dick & Patton, 1992; Hughes-Hallett & Gleason, et al. 1994). The derivative can be represented numerically, algebraically and geometrically. Hughes-Hallett, Gleason, et al. (1994) refers to this notion as the Rule of Three. The fourth component that is added is application. An example of application is being able to apply the knowledge you have to solve a problem. This fourth component has been added to the Rule of Three because many textbooks have problems that apply the derivative, so this component was added to see whether the students were able to complete a problem where they had to apply their knowledge. This section focuses on showing four representations of introductory ideas of derivatives using the Rule of Four. Each of the representations below should be considered as an individual problem. All four representations are of the same problem. The algebraic, geometric, and application representations are shown using the equation $f(t) = -16t^2 - 22t + 220$. That equation is not needed to solve the numeric representation.

Numerically.

Finding the derivative of a table of values numerically would only give you an estimate of the derivative for that equation. If a student were given a table, the table shows the values of a function. For example, the student is presented with the following table:

t	0	.5	1	1.5	2	2.5
F(t)	100	96	84	64	36	0

The student might be asked to estimate $f'(2)$, which means estimating the first derivative where the value of *t* is 2. In order to do this, the student would take the slope of a value below and above 2, which results in: $\frac{0-64}{2.5-1.5} = \frac{-64}{1}$. The two values were subtracted to find the average rate of change between the intervals around the value that was needed, which is 2. This is an approach that a student would use to solve the problem numerically.

For the next three representations, the position function is used; which is applied when one has to find the velocity of objects falling. The function, $-16t^2 + v_0 t + s_0$, v_0 represents the initial velocity of a given object, with s_0 standing for the initial position of the object. The position function helps determine the position of a free falling object over a period of time (Larson & Hostetler, et al. 1994) on earth[2]. Many students in calculus study this function. In calculus, when working with real world problems, the velocity of many objects is needed. In order to find the velocity of such objects, this function must be used.

Algebraically.

When a student is given the equation, $f(t) = -16t^2 - 22t + 220$, he/she may be asked to find $f'(2)$, algebraically. To find the derivative algebraically, the student can use the power

[2] This equation takes earths gravity into account.

rule. For the scope of this study, only the introductory topics in calculus are investigated. There are other ways to find the derivative of this equation; however, this study uses the power rule, for this is within the limits of my study. The power rule is defined by Steward (1997) as "$\frac{d}{dx}(x^n) = nx^{n-1}$, if n is a positive integer" (p. 193). For the problem above, since the exponent is 2, the exponent is multiplied by the leading coefficient, which is -16. Then, you would subtract one from the exponent, which gives a result of 1; thereby, leaving the final result of $-32t$. The same thing should be done with the $-22t$. One would multiply the exponent of 1 by the leading coefficient of -22, since you subtract the exponent of 1 by 1, leaving 0; thus resulting in an answer of -22. Any variable or constant to the 0 power is 1. If the power rule was used on the equation above, the answer would be $f'(t) = -32t - 22$. The next step would be to find $f'(2) = -32t - 22$, and substitute 2 for the variable t, which would leave $-32(2) - 22$. The correct answer is -86. For a justification of how the power rule actually gives the derivative see Hughes-Hallett, Gleason, et al., 1994, p. 192.

Graphically.

When the derivative of the function is positive, the tangent of the function is sloping up over that interval; when the derivative of the function is negative, the tangent is sloping down over that interval. If the derivative of the function is equal to zero over that interval, then the tangent is horizontal everywhere and so the function is constant (Hughes-Hallett, Gleason, et al. 1994). Therefore, the sign of the derivative tells you whether the graph of the function is increasing or decreasing. This will help give a sketch of the graph of the derivative of a function.

Application.

Applications of derivatives are important because they can help a student see the practicality of learning derivatives. Stewart (1997) writes, "A ball is thrown straight down from the top of a 220-foot building with an initial velocity of -22 feet per second. What is its velocity after 3 seconds?" (p. 141). In order to solve this equation, the student has to use the position function, $-16t^2 + v_0t + s_0$. From the given information, the new equation is $f(t) =$ -$16t^2 - 22t + 220$. In order to find the velocity after 3 seconds, the new equation is this $f(3) =$ $-16t^2 - 22t + 220$. Now that the student has created this equation, he/she can solve the problem algebraically or graphically (as shown previously in this section). The student cannot use the numerical method because that will only get an estimate and the problem asks for an exact answer. In an application of derivative problems, the derivative is the same thing as velocity.

It is important to note that derivative means velocity when time is the independent variable and height is the dependent variable. But when velocity is the dependent variable instead of height, then derivatives of that function means acceleration. The definition of derivative is broader than the way this application problem defines it. So far, everything that has been written here about the derivative assumes time and height. This is just one example of what this all means, and this example is one that is commonly used when introducing the definition of derivative.

The research that is related to this study has just been discussed, and derivatives and the different representations of the derivative have been defined. The next section of the study discusses the background, methods used to conduct this study, and a description of how the data has been analyzed.

RESESARCH METHODS

Overview

The purpose of this study is to compare and contrast students' descriptions of their past mathematics experiences with respect to how it impacts their current calculus experience. In their prior mathematics experiences, one group of students chose to take the Traditional sequence (UCSMP) and the other group chose to take the Integrated (CPMP) sequence. The primary foci of the study include examining: a) students' performance on a set of problems involving various representations of derivatives, b) their expressed confidence when solving those problems with respect to various representations of derivatives, and c) their perspective on how their past mathematics classes prepared them (or not) for their current calculus course. In this section, the methodology background of the school and participants, data collection methods, and analysis procedures are explained.

Methodology

Qualitative research attempts to understand and make sense of specific phenomena from the participant's perspective (Reichardt & Rallis, 1994). All qualitative research is interested in how meaning is constructed, how people make sense of their lives and their worlds. The primary goal of a basic qualitative study is to uncover and interpret these meanings (Merriam, 1998). There are different approaches, however, to attaining this goal. One of these approaches is the interpretative qualitative approach. This type of approach can be defined as "learning how individuals experience and interact with their social world, the meaning it has for them" (Merriam, 1998, p. 4). This study seeks to discover the perspectives of students from different curricular materials histories, so interpretative qualitative research methods are more appropriate because they allow researchers to capture the students' experiences and see the meaning it has for them.

All qualitative research is characterized by the search for meaning and understanding that would provide a rich description (Merriam, 1998). Merriam (1998) defines a rich description as "words and pictures rather than numbers used to convey what the researcher has learned about a phenomenon" (p. 5). Common methods to conduct a study using the interpretative qualitative include interviews, observations and document analysis (Merriam, 1998), all of which were used in this study.

Background

School.

This research was conducted in a high school in the Midwest in a community where a major university is located. The students attending this school are mostly middle class and only a small percent come from lower-and working class backgrounds. The students are primarily White with Asians making up the highest minority percentage of the students attending the school (Lubienski, 2004) The high school was purposefully selected for this study because it maintains a two-sequence mathematics program (Traditional and Integrated sequences), with the choice of sequence beginning in the middle school for accelerated students. The school district has experienced some controversy in the past about its adoption of a middle school *Standards* –based curriculum (see Lubienski, 2004), which has resulted in a lower number of students in the Integrated sequence. At the time of the study, there were two sections of calculus being offered at the participating high school. Of the total students enrolled in calculus during the 2004-05 school year, 46 completed the Traditional sequence and only seven students completed the Integrated sequence. Despite the school district's efforts to promote the Integrated sequence, relatively few students chose to enroll in the Integrated sequence prior to calculus (Lubienski, 2004).

Participants.

The students involved in the study were either enrolled in the Traditional or Integrated sequence prior to their senior year of high school. The students were required to return a parent-permission form before they were allowed to participate in this study (see Appendix A). Even if a student's parent said they could participate, each student still had the option to not participate. Whether students chose to participate or not, the teacher offered a few bonus points on a quiz as an incentive to return the permission forms. All of the students who came from the Integrated sequence chose to participate. Purposeful sampling was used to obtain the participants used in this study so the comparison would be a cross of students of the same ability. The teacher match the students based on their current grades and performance in the classroom. Lubienski (2004) conducted a study, which found that there were no academic differences in the students in the two sequences, as a whole, in terms of algebra readiness test scores. However, there is no way to be sure if this is true of the twelve students in this study. This helps support the idea that the differences found among the students were not about general intelligence, but about the type of learning these kids have been doing. All of the Integrated students at the school chose to participate, so the list of Traditional students who chose to participate were narrowed down so that the Traditional students' grades attained matched those of the Integrated students. Esterberg (2002) describes this kind of sampling as intentionally choosing a participant for specific perspectives they might have. The teacher was then asked to suggest a matching student for each Integrated student from the Traditional sequence based on grades attained and gender[3]. The final group chosen included a total of 12 students, 6 students (1 girl and 5 boys) from the Integrated sequence and 6 students (4 girls and 2 boys) from the Traditional sequence.

Table 1. Distribution of Students by Achievement

	Traditional Sequence	Integrated Sequence
High Achievement (as measured by the teacher's grading system)		
	Susan	Arthur
	Emma	Ben
	Patrick	Jim
Average Achievement		
	Ken	Ron
	Roberta	Matt
	Michelle	Monica

Data Collection

Observations.

Before conducting the interviews, the classroom was observed while the teacher taught the students the topic of derivatives. This observation was helpful in understanding what the students might have been referring to when they discussed their current experience in calculus. In other words, they were observed to provide a context for the students' interpretation because students' experiences in calculus class more than likely influence the way they talk about and understand the concept of derivatives. The observation period lasted six weeks during the fall of the school year. During these observations, field notes were taken and classes were audio recorded with permission from the teacher. They were then

[3] Gender was not the priority over attained grades; in some cases, the grades of the two students were chosen over gender. The teacher felt matching by gender would not make their attained grades compatible.

transcribed by the researcher. It was important to audio record and transcribe the observation notes in order to compare the kinds of questions the Integrated and Traditional students were asking in class. The students, however, did not seem to ask many questions during class.

Interviews.

Two interviews were conducted with each student over the course of two weeks, totaling 28 interviews, during their free period[4] of the school day. All interviews were conducted in the school's counseling office. The setting was fairly formal and provided a quiet place to interview each student individually. The interviews began with a review of the purpose of the study so that they understood the study goals. The students were also told that they did not have to answer some of the questions if they chose not to. All of the students had come to the interview with the worksheet already completed. Their papers were not graded prior to the interview. However, because the solutions to the questions were known in advance, it allowed the researcher to probe further into their thinking when they got answers both wrong and right. The purpose of these interviews was to see how they understood the concept of derivative. The interviews were semi-structured with the goal of exploring a topic more openly and allowing interviewees to express their opinions and ideas in their own words (Esterberg, 2002). This interviewing process helps the researcher to understand the perspective of the interviewee.

Although each student was asked the same broader questions, some students who gave short responses were then probed for clarification or for more information. When students made comments that seemed to have had nothing to do with the question they were asked, they were then asked to elaborate on ideas they were expressing. Sometimes, they were probed for more information when the statements they made were not clear. With the students' permission, notes were taken about their responses during the interview.

The purpose of the first interview was to ask questions regarding the first worksheet as well as pose questions to examine students' confidence when solving the various representations of derivatives. Another purpose of this interview was to get their perspective on how the students' past mathematics classes might have influenced their current calculus experience (See Appendix D). They were specifically probed to find out whether students felt that their past mathematics courses prepared them to be successful in their calculus course. It is important to determine whether, from the students' perspective, they feel like they were adequately prepared throughout high school for calculus. Students are the ones taking these courses, so once they give their opinions on whether the past has or has not adequately prepared them; the teachers can then analyze things going on in these classrooms that can better suit the students.

The goal of the second interview was to ask questions focusing on the second worksheet of the various representations of derivatives and to conduct a member check from the interpretations of the preliminary analysis of the first interview. This is further discussed in the analysis section. Subsequent questions about the students' confidence when solving the different representations were asked to examine the consistency of the students' responses. The second interview followed the same format as the first interview. Each interview lasted thirty to fifty minutes in the quiet space of the counseling office. The students were given the worksheet ahead of time to work on and we discussed it along with other questions during the interview.

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[4] Some students have a free period throughout the day in which they go to study hall or can leave campus.

If, from the interview data, the students felt like they were not prepared in their previous mathematics experience, then areas the students were not prepared for were examined in the curricular materials that were used. Although the participants were given the option of not answering any of the interview questions, all fourteen students answered all of the questions to the best of their ability. The interviews were audio recorded with the participants' permission and later transcribed by the researcher in order to capture exactly what the students were saying. In the section related to the data analysis, the process of transcription is more fully described.

Task Worksheets.

To investigate the students' performance on a set of problems involving various representations of derivatives, two worksheets (see Appendix C) were developed that addressed derivatives. In this study, the students were asked to complete a worksheet prior to each of the first and second interviews. The first worksheet consisted of problems asking the students to define derivatives in words, to find the average rate of change, to determine how the equation of a derivative is derived, to find the derivative graphically, and to use the definition of derivatives to solve the problems. This worksheet was created to see how well students understood derivatives. One way of uncovering students' understanding of derivatives is to see if one knows the definition, where the derivative is derived from, and applications of the derivative (Orton, 1983). The average rate of change was added to the worksheet because if a student does not understand that concept, it would be difficult to understand the underlying meaning of derivatives. Some of these questions were adopted from a task worksheet that was developed by Orton (1983). Questions were chosen that included the four representations.

Orton (1983) investigated students' understanding of elementary calculus through the use of clinical interviews. He interviewed both high school and college students about their understanding of derivatives. His main focus was on students' common misconceptions and errors when they are asked to solve a variety of concepts and skills related to differentiation and applications of derivatives. He assessed the problems on a five-point scale, which he created to narrow the focus to a set of numbers. Not only were some of his tasks used in this study, but this study actually goes further in trying to understand more about the students' experiences and how those impact their calculus learning.

Although the focus of both his study and this study differs, the tasks he presents are useful tools for capturing students' understanding of derivatives. In fact, many researchers that study derivatives cite him as a reference because of the amount of work he has done with students' understanding of derivatives. He is viewed as an authority on this topic; therefore, some of his tasks were adopted for this study.

Once the data was collected and analyzed from this first worksheet, it was decided that an additional worksheet was needed to further assess more of the four representations of derivatives, as mentioned in the previous section. The first worksheet only concentrated on the graphical representation of derivatives. A second worksheet was created so that it consisted of problems involving questions with the multiple representations of derivatives[5], including numerical, algebraic, graphical, and application problems because originally the study was designed to examine the students' understanding of the concept of the derivative. However, later on, it was decided that the primary focus of the study should be on the students' understanding of the different representations of derivatives. So, data was drawn primarily from the second tasks worksheet, which focuses on the various representations of derivatives. Some of the students' responses in the first interview led to the insight that the students from the Integrated sequence might perform better on the application and graphical representations than the students who had taken the Traditional sequence, so it became evident that more information needed to be gathered about this. It was also observed that when the Integrated students answered one of the questions where they could chose any representation to use, some used different representations than the Traditional students on the first worksheet. It was expected that the students would solve the questions in the same manner. Since they had not, it became important to see if they all knew how to solve the various representations. If they did it would then be necessary to find out if they learned those ways in calculus or in their previous mathematics classes.

The questions for this second worksheet were adopted from a textbook written by Larson et al. (1994), which emphasizes all four representations of derivative in its problems. Larson et al. (1994) says that, "Every topic should be presented geometrically, numerically and algebraically" (p. vii). They believe that calculus should encourage students to think about the geometrical and numerical meaning of what they are doing, and that this will help students give meaning to symbols. They also see a strong need for application problems because application problems give a student an opportunity to explain their answers in practical terms. Because of their philosophy, the types of problems they offer seem to allow researchers to gauge students' understanding of the different representations of the concept of derivative.

[5] These representations can be solved in more than one way; however I looked for ways to solve these representations similar to those in their calculus textbook.

Each worksheet was given to the students the day before the interview by their teacher to make up for the lack of time allotted for the interview. Many of the students said they completed the worksheet the night before their scheduled interview at home. Only one student had to complete two of the problems during the interview because he did not answer them the night before. The cover letter had instructed them to complete the tasks worksheet individually and directions were given at the top of each section for them to read. During the interview, they were asked if anything was not clear, and all of the students said they fully understood what was expected of them. They were also asked if they had completed the worksheets individually, and all of them said they did.

Data Analysis

In this study, three sources of data were collected and analyzed. Interview data and observations were analyzed to obtain descriptions of students' past mathematics experiences with respect to how it impacts their calculus experience and to examine the confidence of students. The interview data was the primary data source because this study was seeking to get the students' perspectives on their different curricular materials histories. The interviews were analyzed to determine reoccurring themes that arose out of the students' responses. Themes were found between the Integrated and Traditional students' responses separately in order to compare and contrast their themes. This was accomplished by immersion in the data (Esterberg, 2002) through listening to the audio recorded interviews repeatedly in quiet settings. Esterberg (2002) suggests you use various "tricks" to find ways to develop themes, and listening to the data repeatedly proved helpful. After the tapes were listened to multiple times, some open coding was established through working intensively with the data, line by line, identifying themes (Esterberg, 2002) from the transcripts of the interviewees. Themes were developed based on repetition among the student responses. A theme was developed if at least half of the students had the same response. Also, if multiple students stated the same response, it was considered a theme.

The observation data were analyzed after the interviews were conducted by examining the format of the lesson in the classroom while they discussed derivatives. The observations took place to see a sample of the students' current experience in calculus. Although it was hoped that by observing them, differences and similarities in their experiences would emerge from their classroom discussions, the students talked so little that the observations were ultimately not helpful. So, it was decided that the primary focus would be on the kinds of questions students asked (both in terms of the content of the questions and in terms of the type of representations they were asking about) when the teacher took homework question.

The worksheets were a secondary data source used to describe their understanding of the different representations. To do so, the worksheets were analyzed to look at the students' performance on a set of problems, which drew on different representations of derivatives. The first step involved going through each of the problems that the students answered and determining whether they had answered the question correctly. The goal of analyzing the tasks worksheets was to understand the students' solution strategies and their thinking. If a student attempted to answer a question and did not finish it, then that particular problem was discussed during the interview. The solution strategies and correctness of the problems were compared on the tasks worksheets between the Integrated students and Algebra students' responses.

Trustworthiness of a study has to do with issues of validity and reliability. Following the suggestions made by Merriam (1998) to ensure reliability and validity of the study, multiple sources of data collection methods were used to confirm emerging findings. Peer reviews of emerging results were also conducted. A Ph.D. mathematics education student and major professor familiar with the study performed open coding on the interview data. They looked for themes before being told about the themes that were found in order to make sure those themes were apparent in the data.

RESULTS

In this section, the findings related to the research questions posed in the introduction are presented. To serve as a reminder to the reader, the following questions are addressed:

1. How do the students describe their past mathematics experiences with respect to how it impacts their calculus experience?

2. How do the students perform on a set of problems involving various representations of derivatives?

3. How confident do the students say they are when they are asked to solve a set of problems involving various representations of derivatives?

After reporting the findings for these questions for each subgroup of students (i.e., students from a Traditional experience and students from an Integrated curriculum experience), the similarities and differences between the two subgroups of students, who are now taking calculus together, are discussed.

Students' past experiences with respect to their present calculus experience

In this section, both prevalent and interesting themes that emerged in the data analysis are discussed. This section begins with a report on the themes found for the Traditional students, and then addresses the themes found from the Integrated students' data.

Traditional students.

The themes found among the Traditional students' data include their a) thoughts about the phases of a lesson and their preferences related to those phases, b) lack of concern with understanding the mathematics content, c) lack of familiarity with derivatives prior to calculus and d) inexperience with explaining concepts.

Phases of a lesson and their preferences. Mehan (1979) described the structure of classroom lessons as consisting of three phases: an opening, an instructional phase, and a closing. When the students were asked about how their past mathematics course helped them in calculus, the Traditional students spontaneously described their thoughts about the phases that took place in their calculus classroom in comparison to their past experience. This theme can be seen in the following quotes from the interview data:

> "The class is pretty much just like my other classes; he goes over the homework and asks if we have any questions, then he lectures the new material, assigns us homework for that night, then lets us work independently on homework in class." (Roberta, TS, Interview1)[6]
>
> "...the set up of the classes are the same as my other classes...he lectures and gives homework." (Ken, TS, Interview 1)
>
> "The set up of the class is similar to what I am used to, I think calculus is based on the Algebra sequence as far as the teaching. Everyday he explains the work, then we do homework problems." (Patrick, TS, Interview 1)
>
> "...he lectures, asks if we have any questions about the new stuff he covers, then he lets us work on our own." (Susan, TS, Interview 1)

All of these students described the phases of the lessons in calculus as being similar to what happened in their past courses. These phases that the students described are what the students are most familiar with in comparison to their past mathematics classes. The students described the phases of lessons in the calculus classroom as: the teacher answers questions

[6] In the parentheses I include: a) the pseudonym of the student who is speaking; b) TS indicates that the person is a Traditional students and IS refers to an Integrated students; and c) whether the comment was made in the first or second interview.

from the students, he then lectures, and allows time for them to work on the homework that is due the next day. Smith (1996) describes this format as teaching by telling.

All of these comments about the phases of the lessons aligned with what was observed in the calculus classroom. On most days, the teacher was observed opening up the class period by asking the students if they had questions from the problems they were assigned the night before. He then instructed the students to go to the next section in the book and presented the material to the students. Next, he would answer any questions the students had which allowed him to clarify the material he presented. Finally, he would close, and allow the students to work either in small groups or individually on the problems that were previously assigned.

Some of the students that talked about the phases of the lessons then continued to talk about their preferences related to these phases: only one student discussed how these phases related to his past experiences:

> "I *prefer* this type of instruction and the way things are taught." (Susan, TS, Interview 1, emphasis added)
>
> "I *prefer* things straightforward, one thing at a time, the teacher teaches in an outlined way. I know what we are going to be doing every day." (Roberta, TS, Interview 1, emphasis added)
>
> "I *like* the way Mr. C teaches the class, it is what I am used to." (Patrick, TS, Interview 1, emphasis added)

More than half of the students that described the phases they spoke of earlier said that they prefer this type of instruction. Just one of these students, Patrick, stated the reason he preferred this type of instruction in calculus: it was the type of instruction that he experienced in his past courses.

Students' lack of concern with understanding the mathematics. More than half of the Traditional students seemed unconcerned with gaining an understanding of the mathematics they were studying. The students focused more centrally on being able to complete the problems:

> "I am not concerned with understanding; I just want to do the problems." (Ken, TS, Interview 1)
>
> "I am not that concerned with really understanding the derivatives, I just want to do them." (Roberta, TS, Interview 1)
>
> "I don't think you need to fully understand something in order to do it, so I don't need to fully understand derivatives." (Patrick, TS, Interview 1)

Many of these comments came after the students were asked if they felt they understood derivatives. One student attributed this to the fact that, from her perspective, the teacher did not focus on having them understand the mathematics: "…he [the teacher] doesn't ask questions that would want me to really understand what is going on, as long as I do the problem right, I will be okay." (Michelle, TS, Interview 1) From these comments in the interviews, it is possible to conclude that these students give priority to answering the problems rather than wanting to focus on understanding the mathematical content.

Familiarity of derivative prior to calculus. In the first interview, the students were asked how much they knew about derivatives before taking calculus this year. This question was asked because during the classroom observations, Mr. C. made a comment in class about the Integrated students studying derivatives in their Integrated sequence. He said, "We are going to cover derivatives, those of you who took Integrated should know a little about what I am talking about." It then became important to know if the Traditional students also considered themselves to be familiar with derivatives.

Five of the six Traditional students reported that they had never heard of derivatives before taking calculus:

> "I didn't know what it was before taking calculus." (Roberta, TS, Interview 1)
>
> "I knew nothing, absolutely nothing [about derivatives]. I started from zero." (Michelle, TS, Interview 1)
>
> "I never heard of it, people used it, but I didn't take the time to understand what it was." (Patrick, TS, Interview 1)
>
> "I never heard of it before." (Susan, TS, Interview 1)
>
> "Nope, [I] never heard of it." (Emma, TS, Interview 1)

Only one of the six students reported that he knew some of the content from outside of mathematics that he learned was related to derivatives: "I learned a little bit of it in Physics. We studied velocity and acceleration, which ties into the instantaneous rate of change." (Ken, TS, Interview 1)

Because of what the teacher said in the classroom, and after hearing that the Traditional students *never* heard of it before calculus, the question of whether or not the Integrated students were at an advantage because they had worked with derivatives prior to coming to this class became very important. Nevertheless, all but one of the Traditional students reported having no previous experiences with derivatives before calculus.

Not "used to" explaining concepts. Half of the Traditional students said that because of their past courses they were not used to explaining their answers. However, not many students pointed out that this is something they are now being required to do in calculus. Only one student, Emma, stated that Mr. C required them to explain their answers in calculus.

> "I am not used to explaining what the answer means in the problem, I am used to just answering the problem and circling it and moving on." (Roberta, TS, Interview 1)
>
> "I am not used to writing out an explanation for my answer that is why I took the Traditional sequence, we had did that in *MiC* [i.e., *Mathematics in Context*, the middle school mathematics curriculum they used]." (Michelle, TS, Interview 1)
>
> "We didn't have to explain what things meant, we just had to do them in my other class." (Emma, TS, Interview 1)

The first two students, Roberta and Michelle, stated that they were not used to explaining their answers in a mathematics class. Michelle even said that she took the Traditional sequence to avoid having to explain her answers. Being able to explain answers shows that you really understand what the answer means, and these students indirectly stated that their past courses did not require that. Emma agreed with the other two students in that her past courses did not require her to explain her answers, but she went on to say that in calculus, Mr. C requires the students to do this. Most of the quotes reveal what students are "used to" doing, but almost all of them do not necessarily compare this to what they are currently required to do.

During the same interview, the students were also asked to explain what the answer meant to problem #5 on the first task worksheet. The same students who are quoted above were able to solve the problem correctly, but they could not explain what the answer meant to the problem. They gave the following responses to this question:

"I don't know how to explain what that answer means, and I usually don't try to do that." (Roberta, TS, Interview 1)

"I can solve the problem, but I cannot explain what it means." (Emma, TS, Interview 1)

"I can't really explain what the answer means, but did I get it right?" (Michelle, TS, Interview 1)

The students were not asked if they thought their lack of ability to explain a solution was related to the fact that they were not required to do so in their past experiences. However, it was important to emphasize that the exact same students who said that they were not used to explaining their answers also said that they could not explain their solutions. Some of the other students made an attempt and answered it incorrectly, but these students did not even try to explain what the answer meant.

When the students asked questions during the observed classes, none of the students in this study asked the teacher to explain what an answer meant to a problem. Many of the students' questions were either about clarifications of the concepts being explained or were asked to help them with problems from the homework. The students were neither asked to explain what the answers meant to the problems nor were they asked to articulate what they were thinking, what they did or why. Also, in their textbook, there were a few assigned problems that required the students to explain either of the two described above. However, Mr. C could have asked them to explain what the answer means to the problem or have the students explain what they did and why on exams or quizzes. Whether the teacher required them to explain their answer is important, but for this study, their past experiences were the focus, and not how their past experiences relate to their present demands.

Integrated students.

The themes that emerged from the Integrated students' interviews include a) descriptions of the phases of a lesson, b) their ability to adjust to the phases of a lesson, and reasons for the adjustments, c) their familiarity with the graphical representation of derivatives prior to calculus, d) their awareness of the type of problems calculus class lacks (which are the same types of problems they prefer to do), and e) their desire to understand the mathematical ideas being studied.

"Adjusting" to different phases of mathematics lessons. While the Integrated students also described the same phases as the Traditional students, they spent more time talking about the *differences* between the phases of the lessons in calculus in comparison to the phases of the lesson in their Integrated classes:

> "Calculus is so different than Integrated, like the way he lectures, then asks if we have questions, then assigns homework." (Monica, IS, Interview 1)
>
> "…the Traditional class would have prepared me more for calculus because of the set up." (Ron, IS, Interview 1) When asked for clarification in the follow-up interview, Ron made the following comment "…the style of the class is different from Integrated. [In calculus] he lectures a lot and asks for questions from the homework, then lets us work after he teaches something new to us." (Ron, IS, Interview 2)
>
> "In calculus, the teacher comes in and answers questions from the work the night before, explains the new stuff, then we spend class time working on the assignment for the night, this doesn't happen in my Integrated classes." (Matt, IS, Interview 1)

Half (three) of the Integrated students spontaneously described the phases similar to those above by describing how they noticed a difference in the phases of the lessons that took place in calculus in comparison to the phases of the lesson in their Integrated classroom.

The students commented on the phases that took place in their calculus classroom and, without being prompted, discussed that the phases of the lesson in calculus were different from the phases of the lesson in their Integrated classes. Two of the Integrated students described some differences between their experience in calculus and the Integrated classroom.

> "[In Integrated there was] a lot of group work and self exploration, learning concepts on my own." (Arthur, IS, Interview 1)
>
> "[In Integrated the] teacher was there to guide you… it [Integrated] seems more interactive." (Monica, IS, Interview 1)

In these two quotes, you can see the description of working in groups and the role of the teacher as one who is there to guide students' learning. Even though the students described earlier that the teacher lectures, they did not mention that they were guided by the lecturing. One student speculated that if he had taken the Traditional sequence, he would have been more prepared for calculus in terms of the phases of the lessons. In the follow up interview, the students were asked which type of instruction they preferred, and both of the students quickly replied, "The Integrated way." (Arthur, IS; Monica, IS, Interview 2)

Half of the Integrated students recognized they needed to adjust to the new phases of the lessons they were encountering in calculus, without being prompted by an interview question.

"I have adjusted well, seeing the types of classes are a bit different…I had to adjust to this kind of teaching style." (Jim, IS, Interview 1)

"There are some differences between the two types of classes… I had to learn to adjust pretty fast." (Ben, IS, Interview 1)

"Even though the classes are not totally the same, I have adjusted to how we do things in calculus." (Matt, IS, Interview 1)

"It was hard adjusting to the new format of how the teacher teaches, but now that I have adjusted I am doing quite well." (Ron, IS, Interview 1)

While all of these students talk about adjusting, the first two seem to indicate the adjustments were pretty small, the second does not really talk about it in terms of scale, but the last seemed to think it was a bigger transition. Not only do the Integrated students have to learn the new, rigorous topics of calculus, but they also have to be able to "adjust quickly", as some of the students said, to the new style of teaching.

Of those three students that discussed adjusting to the phases, two of them spontaneously expressed their reasons why they felt they needed to adjust: "My calculus class is how college math is taught." (Jim, IS, Interview 1) "I would choose the way calculus is taught because it prepares me for how they do it in college." (Ben, IS, Interview 1) These students assume that mathematics teaching at the college level is aligned with the phases of the lessons from their calculus class. They are looking ahead and figure that they need to adjust to this new style of instruction in calculus in order to prepare for the style of teaching they will experience in college.

Familiarity of derivatives prior to calculus. Because of the comment Mr. C made about the Integrated students being familiar with the word "derivative", it became important to see what the Integrated students would say about what they knew about derivatives before entering calculus. While only some of the responses were the same among the Integrated students, in that they discussed the concept of derivatives graphically before calculus, each of the students said s/he was familiar with at least one topic related to derivatives:

> "I could tell which graph was the derivative when given two graphs, where one slope was zero; the derivative would pass through the x-axis at that point." (Jim, IS, Interview 1)
>
> "I was familiar with some of the rules, like the power rule and the quotient rule." (Ben, IS, Interview 1)
>
> "We found the instantaneous rate of change and graphed the derivatives." (Ron, IS, Interview 1)

Both Arthur and Matt said they were familiar with graphing the derivative using their calculator. The majority of the students said they were familiar with finding the derivative graphically. It was expected that the students would have had some experiences with the derivative, but it was not expected that they would have already examined one representation of the derivative before entering calculus. Their performance on the graphical representation of the derivative is discussed later in the results section.

Students' preferences about the types of problems lacking in calculus. The students discuss the types of problems that do not occur in calculus:

> "In Integrated we see things differently because the word problems are more real life and we work towards finding the rules; we don't do hardly any word problems in calculus" (Michelle, IS, Interview 1)
>
> "We don't do many word problems in calculus." (Ron, IS, Interview 1)

"We don't do a lot of word problems in calculus; I wish we would do more." (Monica, IS, Interview 1)

"In calculus, we don't concentrate on word problems, where we apply what we learned, like in Integrated…" (Matt, IS, Interview 1)

"In Integrated we did a lot of word problems where we applied the math we learned, we don't do those kinds of problems in calculus. (Ben, IS, Interview 1)

"In calculus we do less story problems compared to my Integrated classes." (Arthur, IS, Interview 1)

All of the Integrated students said that in calculus they do not do many word/story problems now (as often as they would like). However, only some expressed their dissatisfaction with that. The first student not only comments on the fact that they do not do many word problems, but she also connects word problems to making mathematics more connected to real life. One student connects the use of word problems as the opportunity to apply what it is they learned. Lastly, many of the students compare the number of word problems done in calculus and their experiences in their Integrated class.

Half of the Integrated students reported that they either found those types of problems interesting or they really enjoyed working on those types of problems. These comments came from the students after they worked on the last problem on the second task worksheet. In that problem, they were asked to apply the information to form an equation and solve the problem: "Given the position function, $-16t^2 + v_0t + s_0$. A ball is thrown off a 270-foot building with initial velocity of -22 feet per second. What is its initial velocity after 3 seconds?"

"Those [word problems] are most interesting to me to do." (Jim, IS, Interview 2)

"It was refreshing to see a problem that I had to apply the information to do the problem…I like doing those problems." (Ben, IS, Interview 2)

"I like doing word problems because I have done a lot of them in Integrated." (Matt, IS, Interview 2)

Even though the students see that their calculus class lacks word or story problems, they still expressed an enjoyment for working on those types of problems. At least one of the students, Matt, related this enjoyment to the fact that he did a lot of these kinds of problems in his Integrated courses.

Desire to understand the mathematics. The last theme that will be discussed involves the Integrated students' desire to understand the mathematics they do. The majority of the Integrated students expressed that they would like to know where the formulas and rules come from and said that knowing this information helps them to understand the mathematics better. For example:

"I wish we would go over where formulas come from [in calculus]…it would help me understand what I am doing." (Jim, IS, Interview 1)

"I'd hope we concentrate on where those rules and formulas came from, that would give me a better understanding" (Monica, IS, Interview 1)

"We have to know the rules and formulas, but seeing where they came from would help me understand more of what is going on." (Ben, IS, Interview 1)

"I wish we had problems where I knew where the rules I use[d] came from, it would help me understand better." (Ron, IS, Interview 1)

"I want to understand where the formulas come from, not just use them. Knowing where they come from would help me understand the math better." (Arthur, IS, Interview 1)

The Integrated students were concerned with understanding mathematics and they believed that knowing more about the rules and formulas, such as "where they come from," would aid in this understanding. Both the high and average achieving students felt this way. So, even though the students were performing well, they still "wished" the class would concentrate on this aspect of their learning.

Performance on the various representations of derivatives

This section discusses the performance of the Traditional and Integrated students on the various representations of derivatives. The first issue to be discussed will be whether the students answered the four problems on the second task worksheet correctly or incorrectly. As stated earlier, this study draws heavily from the second worksheet because it was geared towards the various representations of derivatives. Then, information is provided about whether the students justified their answer and describe the nature of those justifications.

Traditional students.

The majority of the Traditional students answered correctly at least three of the four problems. Many of the Traditional students answered the numerical approach using the slope formula. All of the students reported that the power rule was the easiest to solve. All of the students answered the graphical representation problem correctly; however, they approached the problem differently. More than half of the Traditional students did not attempt to answer the application problem.

Numerically. Five of the six students answered the numerical problem correctly. Of those five students, four of them (Emma, Michelle, Patrick and Susan) showed their work. In their written justifications, they all wrote that they "used the slope formula by looking one below and one above the number I needed to estimate which was 2." Ken, the other student that answered the problem correctly did not show his work. When asked in the interview why he did not show his work, he stated, "I do not like to show my work, I rarely do in my class and I lose points because of it." (Ken, TS, Interview 2) When asked if he could verbally say what he did, he stated, "Well, since I had to estimate 2, I took the slope of the values one above and one below." Rachel was the only student who did not answer or attempt to answer the problem at all. When asked why she did not answer the problem, she said, "I did not know how to do it." Not only did the students that answered the question answer it correctly, but they also gave thorough explanations of how they solved their answer. Given the thorough explanations of their process to solve the problem, follow up questions included whether they were confident in their responses.

Algebraically. All of the Traditional students answered this problem correctly. They all used the power rule and then substituted the number 2 for the answer. When asked why they used the power rule, the students all had the same response, "Because it was the easiest way to solve that problem." The students could have solved the problem graphically or used the definition of derivative; however, it is suggested that when asked to find the derivative of a polynomial, it is faster to find the derivative using the power rule. (Dick & Patton, 1992) From the previous interview, it was assumed that they would all know how to answer this question and it turned out that this assumption was correct.

Graphically. All of the Traditional students correctly graphed the equation and found the derivative. All of them said that they used their graphing calculators to help them solve the problem, but they used them in two different ways. Ken, Emma, Roberta, and Patrick found the derivative of the equation using the power rule. Then they graphed the equation on their graphing calculator. Finally, they sketched a picture of that graph on their task worksheet. Although it was known that this was a way to solve the problem, it wasn't anticipated that the students would solve the problem in this way. When the four students were asked why they chose this strategy to solve the problem, they all said, "That is the easiest way to do it." As mentioned earlier, when one has a polynomial, a simple way to find the derivative is using the power rule. Given the fact that this method wasn't one that the instructor was observed using, it was surprising that the students used it.

The other two students, Rachel and Susan, solved the problem in the way the instructor taught them to do it in the classroom. They graphed the equation on their graphing calculator, then drew out a sketch of that graph on the task worksheet. They then sketched out a graph based on these criteria: If $f(x)$ has a positive slope, then $f'(x)$ will be positive in that interval; and if $f(x)$ has a negative slope then $f'(x)$ will be negative in that interval. If $f(x)$ is at 0, then $f'(x)$ is at 0. Only two students, Rachel and Susan, mentioned the maximum and minimum in their answers. Even though different approaches were used to solve the equation, all of the students answered the problem correctly.

To become efficient problem solvers students need to understand when and how to use derivative formulas and methods. These two groups of students chose to use a method that is comfortable and familiar to them. Problems that have different representations allow students to use different approaches to solve the problems. When answering a problem that asked them to "find the derivative graphically", the second group of students solved the derivative graphically according to their textbook, while the first group solved the problem using a familiar method that was taught by the teacher.

Application. Only two out of the six students answered the last problem correctly. The other four students did not even attempt to do the problem; they left that part of the task worksheet blank. Ken and Patrick were the two students who answered it correctly. They also used the same strategies to solve the problem. Their justifications of how they answered the problem were very similar:

> "The key thing in this problem is knowing what number to substitute in the problem. V_0 is the initial velocity, so I had to substitute the -22 and S_0 is the initial position of the object. Once I knew that I just used the power rule to do the rest. This was really easy." (Ken, TS, Interview 2)
>
> "All I did was substitute the initial velocity and the height of the building in the problem. If two numbers are given in the problem and two missing numbers are needed, I would automatically think those two numbers need to go into the equation. That formula is in my Physics book, so I knew which one was the initial velocity. Then I used the great power rule, this problem was easy." (Patrick, TS, Interview 2)

The students were right: the key strategy was to recognize which number was the initial velocity and then substitute the correct numbers. Once they have figured that out, they can use the power rule to solve the equation graphically. Both of the students agree that using the power rule was the easiest way to finally complete the problem. The students initial starting points were different. Ken knew immediately to substitute the initial velocity and position of the object. While Patrick did not see immediately which numbers needed to be substituted, he then attributed his Physics class experience (rather than his calculus class) to helping him know how to solve the problem.

The other four students did not attempt to answer this application problem. The reason for the lack of response varied from hating word problems to not knowing where to start:

"I hate word problems, I never liked them." (Susan, TS, Interview 2)

"I didn't know where to start" (Roberta, TS, Michelle, TS, Interview 2).

Emma attempted the problem, but she incorrectly substituted the numbers in the equation. When I told Emma her answer was incorrect, she stated, "I have no idea how to do this problem" (Emma, TS, Interview 2).

Integrated Students.

The majority of the Integrated students answered at least all of the problems on the second tasks worksheet. All of the Integrated students answered the numerical, algebraic and graphical representation problems correctly. At least half of the Integrated students answered the application problem correctly.

Numerically. All of the Integrated students answered the problem involving the numerical representation. They all had reasons similar to this one: "I knew I had to estimate the derivative at two, so I chose a value one below two and above two. I then used the slope formula." (Jim, IS, Interview 2) Matt did not show any work: he just wrote the answer. When asked about this, however, he said he worked the problems on a different sheet of paper and forgot to rewrite his work. The students weren't expected to use different strategies to solve this problem because there is only one way to solve it.

Algebraically. All of the Integrated students solved this problem correctly using the power rule. The students were asked how they knew to use the power rule and all of them had similar answers. For example, "When you are taking the derivative of just an equation you use the power rule. We did a lot of these in the classroom." (Ben, IS, Interview 2). Many of the students agreed that that was the easiest problem on the worksheet.

Graphically. All of the Integrated students answered the graphical representation correctly. They were asked how they first started to solve the problem and all of them said that they used their graphing calculator because it would be quicker and easier than trying to sketch the graph by hand. All of the students graphed the equation on their calculator, then drew it on their worksheet and used a justification similar to this response: "When the derivative is zero, then the function is at zero. I used that as my starting point. Next, I looked at which intervals of the derivative were increasing. I knew the function would increase also, and if the derivative interval was decreasing then the function would decrease." (Ron, IS, Interview 2) All of the students used the zero as their starting point.

Application. Four out of six of the Integrated students answered the application problem correctly. Two students used the same approach while the other two students used different approaches. Another student attempted the problem, but had the wrong answer. Only one student did not attempt to do the problem at all.

Two students admitted to me that they used the book to aid them in answering the problem. They both said they looked up the position function in the index and found that my problem resembled a problem given in their textbook. While it was hoped that they would solve this problem without using their textbook, this shows that they know when and how to use their resources when they are unsure about how to solve an unfamiliar problem. Another

Integrated student substituted the numbers into the equation. "I took Physics so I know the symbol for velocity, I substituted that one in and realized that I had one more number and an empty symbol, so I substituted that number in and then I used the power rule." (Jim, IS, Interview 2) The last student used two methods to check to make sure his answer was correct. "I used the power rule once I got my new equation and then I graphed the equation and solved it how I solved number three." (Ron, IS, Interview 2)

Even though four of the students got the wrong answer, there was more variety in the approaches they used compared to the Traditional students. Because of this diversity of incorrect responses, some of the things these students did were delineated because it provides an opportunity for the reader to know that their incorrect answer might not have been attributed to them not knowing how to answer the question.

One student substituted the wrong numbers, but when told that he was wrong, he knew it immediately and wanted to redo the problem. "I should have known that 'v' stood for velocity." (Matt, IS, Interview 2) Once he reworked the problem, he used the power rule after he substituted in the numbers correctly and ended with, "That was easy." (Matt, IS, Interview 2) The last student did not attempt to do the problem and attributed the lack of response to running out of time: "I didn't do the problem; I really didn't have enough time to do it." (Monica, IS, Interview 2) When asked if she wanted to complete the problem during the interview she opted not to, explaining "I do not work well under pressure."

Confidence when solving various representations of derivatives

This section discusses the various representations of derivatives that both the Traditional and Integrated students felt confident solving. The students completed the second task worksheet which required them to solve four questions, using the four representations (i.e., numerical, algebraic, graphical, and application)[7]. During the second interview, they were asked what representation they were most confident in solving. They were also asked to explain why they were confident solving the representations they chose. Many of the students, both Traditional and Integrated chose more than one type of problem.

Traditional students.

Three out of the six students said they were confident in answering all of the problems on the worksheet. In fact, these were the only three who said that they were confident solving the application problem on the task worksheet:

> "I am confident in doing all of the problems. In class we do more problems like #1, 2, 3, but I can do #4 because I take the time and do extra work… I do the word problem in the book when they are not assigned as homework" (Ken, TS, Interview 2)

> "I feel confident doing all of the problems. I have worked a lot of problems on them both when they are assigned and when they are not assigned." (Patrick, TS, Interview 2)

> "I feel confident solving all of the problems. We do a lot of each of these problems, so from doing many of these, I am able to be confident in doing all the problems you have given me." (Susan, TS, Interview 2)

The first two students attributed their understanding of #4 to their hard work outside of their classroom requirements and experiences. Both students said they work on word problems, even when they are not assigned. The first student indicated that they are required to do as

[7] The students made references to the types of problems based on the number it was on the task worksheet; therefore, #1:Numerically, #2: Algebraically, #3: Graphically, #4: Application

many of the word problems in their calculus class. The last student indicated she was confident in solving all of the problems because those are the types of problems they do in the classroom. While being observed, however, Mr. C gave me a list of the problems he assigns for homework, and it was noted that he does assign word problems to the students for homework. It was also observed that Mr. C answered students' questions regarding word problems that were assigned for homework.

Of the six students, one student felt confident solving derivative problems only algebraically and attributed this to working on these types of problems a lot in her calculus class:

> "I am most confident solving a problem using the rules [algebraically], I can just apply the power rule and substitute. I have done them over and over again for problems in class…I am least confident in [applying the derivative], I don't know where to start. We don't do too many of these types [of problems] in the classroom." (Roberta, TS, Interview 2)

Here Roberta stated that she was least confident solving the application problem because, as other Traditional students said, they do not do a lot of those problems in their calculus class. Roberta is the only student that stated that she is only confident in solving one type of representation. However, as was shown earlier, she was not the only Traditional student that stated that the application problems are the hardest.

Overall, half of the Traditional students were confident completing at least the first three problems. They said they were most confident doing those problems because they do a lot of them in their calculus class:

> "I am most confident doing #1, 2, and 3. They are all particularly easy for me because we do a lot of them in class. (Emma, TS, Interview 2)
>
> "I am most confident in solving #1, 2, 3, because I have a strong algebra background. We have done a lot of these problems in the classroom…I didn't know where to start to do number 4. I am really not confident doing problems that I have to apply the information to a story problem. I was never good at problems like that." (Monica, TS, Interview 2)

Emma and Monica said they were confident in doing the first three problems on the task worksheet because they did a lot of them in their classroom. Monica went on to further attribute her success on the first three problems to her strong algebra background. She then went on to say that she was never good at answering story problems.

Integrated students.

All of the Integrated students stated that they were more confident in answering a problem that has a graphical representation of a derivative. For example, one student said, "I am most confident in doing problems with graphs. We did a lot of problems with graphs in Integrated, so I had a lot of practice before coming to calculus." (Monica, IS, Interview 2) The next two students, Matt and Arthur, were confident doing problems with the representations that include graphically and algebraically. Arthur, however, includes that he is also confident doing application problems.

> "I am especially confident in doing problems with graphs and using power rule because in Integrated we did graphs and worked a lot with power rule in calculus." (Matt, IS, Interview 2)

"I am confident solving a derivative with graphs, using the power rule, and [problems] that applies the derivative. We have done a lot of those problems in my Integrated class." (Arthur, IS, Interview 2)

Notice that these students attributed their confidence in solving the derivative graphically to their Integrated class experience rather than to their calculus class.

Three out of the six students stated they were confident doing all of the various representations.

"I am confident in doing all of the problems. I don't think I can choose one that I am most confident in…we don't do many of these [applying the derivative] problems in class." (Ben, IS, Interview 2)

"I am confident doing all of the problems. I can say though that taking Integrated has allowed me to be confident in doing graphing and story problems." (Ron, IS, Interview 2)

"I am most confident in all of the problems. We did a lot of application and graphing problems in Integrated. And the power rule is just really simple to do. (Jim, IS, Interview 2)

All of the students stated that they were confident doing all of the problems on the second task worksheet. A closer look at the responses individually reveals the following: One student, Ben, said he was confident doing all of the problems, but they do not do many application problems in their calculus class. Two of the students, Ron and Jim, attribute their confidence in doing the application and graphing problems to their Integrated classes. Lastly, Jim said, as many students have said, that the power rule is easy to do.

Similarities between the students

This section discusses the similarities between the Integrated and Traditional students' descriptions of their past mathematics experiences with respect to how it impacts their calculus experience. There were only two similarities between the two groups of students: they both 1) notice the same phases of the calculus lesson; and 2) agree that calculus lacks word or story problems.

Many of the students describe the phases of the lessons that take place in the calculus classroom in the same manner. And, as stated earlier, the phases of the lessons they described were also observed. While not all of the students were happy with the phases (as is discussed in the next section on the differences between the groups of students), all of their descriptions about the particular phases of their calculus class were the same.

Many of the students also agreed that there is a lack of word problems in calculus. Some of the Traditional students attributed their poor performance on the application problems to the fact that they do not do many word problems in class. There were similarities with some of the Integrated students who said that they performed well on application problems because of the amount of them they did in their Integrated class.

Both groups of students seem to agree on two things that happen in their calculus classroom: they agree on the phases of the lessons and on the types of problems that they do in calculus. However, even though they agree on these two aspects of the calculus class, they have different preferences about these things. No similarities existed in the students' descriptions of how their past experiences impacts their current experience in calculus.

Differences between the students

This section discusses the differences between the Integrated and Traditional students' past mathematics experiences with respect to how it impacts their calculus experience. There were many more differences between the students than there were similarities. The differences discussed include the fact that Integrated students cared about gaining an understanding, while the Traditional students did not; the Integrated students preferred to have word problems, while the Traditional students preferred the type of instruction that occurs in calculus; coming into calculus, the Integrated students were familiar with one of the representations of derivatives (i.e., graphs), while the Traditional students were not; and finally, the Integrated students felt like they had to adjust to the type of instruction while the Traditional students did not.

Some of the Traditional students seemed not to care whether they understood the mathematics. All they wanted to do was "do" the problems, and they were satisfied with that. By "do" the problems, it is meant that some of the students are only concerned with being able to solve the problem correctly. The Integrated students "wished" they would get an understanding of the material in their calculus class. They equated gaining an understanding with knowing where the formulas and rules come from.

Both groups of students had preferences related to what occurred and did not occur in the classroom, but those preferences were different. Some of the Traditional students preferred the type of instruction that took place in the classroom. A subset of these students said that they liked this type of instruction because it was similar to what they were used to in their prior mathematics classes. In contrast, when the Integrated students discussed what they preferred, they spoke of word or story problems. There was a difference between the reported need for adjustments in the calculus classroom in regards to the instruction. The Traditional students said that the phases of the lesson are similar to what happened in their past classes. The Integrated students reported that the phases of the lesson are not similar to what happened in their past classes. This lack of continuity in the lesson phases caused the Integrated students to adjust to the differences as well as learn the new material being presented. Some of the students said that they do not mind adjusting to the new lesson phases because they think college mathematics is taught this way and it was allowing them to get ready for that kind of instruction.

Even though both students noticed the lack of word problems in calculus, the Integrated students said they would have preferred to have more of those type of problems. The students did a lot of word problems in their Integrated classes, so it was something that they were familiar with doing. A couple of students made a reference to liking word problems because they related mathematics to real life.

Another difference relates to the students being familiar with at least one aspect of derivatives before taking calculus. The Traditional students admitted they did not know anything about derivatives before taking calculus (with the exception of one student who said that he saw this idea in Physics). However, the Integrated students were familiar with at least the graphical representation of the derivative. Both the Traditional and Integrated students performed well on the graphical representation, so it may be that even though the Traditional students had no previous experiences with the graphical representation, it did not hinder their performance with the representation. Based on observations of the class, the consistency of the success with the graphical representations across the groups of students made sense, given the fact that their calculus teacher spent quite a bit of time on this representation.

The biggest difference between the students' performance on the various representations of derivatives was the application problem. Four of the six Integrated students versus only two of the six Traditional students answered the application correctly. Overall, all of the students performed equally well on the other representations, and he Integrated students seemed to prefer and feel confident solving the graphical and algebraic problems.

Summary

In summary, themes among the Traditional students in relation to their past mathematics experiences include the following: a) thoughts about the phases of a lesson and their preferences of those phases, b) lack of concern with understanding the mathematics content, c) lack of familiarity with derivatives prior to calculus and d) inexperience in explaining concepts. The themes among the Integrated students include the following: a) description of the phases of a lesson, b) ability to adjust to the phases of a lesson and reasons for the adjustments, c) familiarity with the graphical representation of derivatives prior to calculus, d) awareness of the type of problems calculus class lacks (which are the same types of problems they prefer to do), and e) a desire to understand the mathematical ideas being studied. Both groups of students' performed equally well on the algebraic, graphical, and numerical representations. The Integrated students performed better and felt more confident about the application representation than the Traditional students did. The students were confident on the problems that they answered correctly on the second task worksheet.

DISCUSSION

This section I a) compares this study's results to the results reported in the literature review and discusses why its findings are significant to the mathematics education community, b) states the implications of this research for policy makers, teachers, and curriculum developers, c) discusses what lessons were learned through the process of doing this master's thesis and what could be done differently if this study were done again, and d) delineates some of the limitations of the study.

Comparison of this study's results to results reported in the literature review

This study's results align with many of the studies presented about student achievement in *Standards* based curriculum. Thompson and Senk (2003) state that secondary classrooms need to include more problem solving. Both the Integrated and Traditional students in this study claim that their calculus class lacks word and story problems. In fact, the Integrated students prefer doing those types of problems. Four of the six Integrated students answered the application problem correctly versus only two out of the six Traditional students. This study's results correspond to the results of Boaler (1998), who found students who came from *Standards* based curriculum performed better on problems where they had to apply the procedures as compared to students who came from a Traditional curriculum.

Thompson (2001) found that students from a *Standards*-based curriculum outperformed students on problems involving application and graphical representations. As stated earlier, this study showed that the Integrated students outperformed Traditional students. In contrast to Thompson's findings, this study shows that the two groups of students performed equally well on the problem involving the graphical representation.

This study found that Integrated students were familiar with the concepts of derivatives before entering calculus, while the majority of the Traditional students were not familiar with the concepts of derivatives. This corresponds to the results of Schoen and Hirsch (2003a) who found that Integrated IV covers more material in preparation for calculus than Pre-calculus, a course taken by the Traditional students.

Smith and Burdell (2001) found that students who take a reform curriculum are asked to explain their thinking more and are required to work in groups. This study found that students who take a Traditional curriculum are not used to explaining their thinking when solving problems. An Integrated student noted that in his Integrated class they worked in groups frequently.

Hirschhorn (1993) conducted a longitudinal study on students that used the UCSMP curriculum and found that the students outperformed students who did not use that curriculum on problems of application of mathematics. His results contradict what was found in this study. The majority of the students that came from the UCSMP curriculum in this setting could not answer the application problem on the task worksheet and even expressed a lack of confidence about these problems.

As McLeod (1992) stated, much of the current research on confidence has reproduced the same results found in earlier research. Reyes (1984) found that students who are not confident on certain problems will not be persistent in finding the correct answer. Some of the Traditional students in this study were not confident in solving the application problems and many of their responses were "I can't solve those types of problems because they do not do those types of problems in the classroom," or "I was never good doing types of problems." Some research (Dowling 1978; Anderson 1994) has shown that students perform well on tasks that they were confident doing. The same results were found in this study; students were confident on problems that they had answered correctly. This study looked at the different types of representations of derivatives that these two groups of students were confident doing, and found that the Integrated students were confident doing all of the representations, but that the Traditional students were not confident doing the application problems. Instead, this study found that they were confident in the other three representations. It is unknown whether there are any studies further examine this issue.

Lubienski (2004) contends that there needs to be more studies that portray the students' perspective. Smith et al. (2000) reported students' perspective as they transitioned from a *Standards* based curriculum to a Traditional curriculum. Through interviews and observations, they reported that students noticed differences in the types of problems they encountered in the Traditional curriculum; more specifically, the students were referring to word or story problems. This finding correlated with the findings of this study, in that the students from the Traditional and Integrated curriculum noticed a lack of story and/or word problems in their calculus class. They also found that the students coming from a Traditional curriculum to a *Standards* based curriculum were not used to having to explain their answers to the problems. This finding corresponds with my results in that the Traditional students stated that they were not used to explaining their ideas when their calculus teacher asked them to do so. They attributed this to the fact that they did not have to do that in their previous courses.

Through interviews, observations, and a task worksheet, this study gives the students' perspectives of their past mathematics experiences in relation to their current experience in calculus. In observing the students, some of the issues they said occurred in their current classroom were recognized, and what they had to say about their past experiences were taken into account. . This study also gives an overview on the performance of various representations of problems on derivatives. Finally, a task worksheet was examined to see whether their performance aligned with their comments during the interview. For example, many of the Traditional students could not answer the application problem on the first task worksheet, and their perspective was that application problems were hard and that they were not used to working those types of problems.

Smith et al. (2000) stated that research investigating students' perspectives from two different curricula should not "provide evidence for one against another" (p. 126). This is something that those who read this study should not take away from it.. Each of the students in this study had the opportunity to choose the sequence they were enrolled in. For whatever reasons, these students chose these different programs for particular reasons. This research attempted to provide insight for discussions among educators about what is going on, from the students' perspective, in these two curricula. From the students' perspectives, are these two curricula meeting the goals they set forth to meet? It is important to notice whether or not the goals of the curriculum are being met for the students in these curricula in order to decide if changes need to occur.

The students' perspectives should be a key factor in planning and discussing issues in mathematics education. These results show that some students value understanding, which they claimed was part of their past curriculum. The results also show that some students said that they were not exposed in their past curriculums to many word or story problems where they could apply what they have learned. In mathematics, these are essential; more importantly, the students find these to be important.

Implications of this research

Teachers, policy makers, and curriculum developers have something to learn from this study. Not all mathematics can be represented in various ways, but there is a lot of mathematics content that can. Teachers should make sure students engage with and understand the various representations. All mathematics should be geared towards understanding the content, and students should learn how to apply the mathematics they are learning. Some students in this study wanted to apply the mathematics they were learning but felt that they did not have many opportunities to do so.

District level policy makers that decide to offer two mathematics sequences at a high school to accommodate students' and parents' preferences need to think critically about the effect of such a decision. Some positive aspects reported by Integrated students were that they wanted to know where some rules and formulas came from. Also, the Integrated students are being exposed to some calculus topics in their Integrated IV classroom. However, there were also some possible negative implications to allowing a choice in curricular options. For example, the students coming out of the Traditional track did not seem to care if they understood the mathematics. Most policy makers would agree that this is not a good thing. It is very important for students to be able to explain what an answer means to the problem and articulate what they were thinking, what they did and why. It is also important that students care whether they understand the mathematics or not. If we had only used conventional measures to compare students (for example, the methods the teacher used to match the students, like homework scores and other in-class assessments, as well as standardized tests), these students may have looked very much alike. The nature of assessments of students is problematic because it misses students understanding. Different orientations about students' learning may not be apparent. This study helps the reader to go beyond just test score numbers and delve deeper into understanding the students' perspectives about their experiences.

Differences among the students weren't taken into consideration before they started their two sequences. Therefore, it isn't possible to make the claim that they learned these dispositions in their prior sequences. It is a possibility that the students' dispositions were different before they entered sequences. Whether the Integrated sequence can promote the disposition to care about understanding mathematics is an important empirical question for mathematics education researchers to examine.

There will always be an assortment of different curriculum materials in mathematics education. We need students to be able to apply the mathematics they learn in the classroom. It is good to be able to solve an equation; it is also important for students to get the opportunity to read a situation, construct an equation for the situation and solve it. Curriculum developers need to make sure the type of curricula they develop include these types of problems. And more importantly curriculum developers need to include questions that are geared towards getting students to explain their thinking by articulating what they did and why they did it. A stronger bridge needs to be made between skills and applications. This study shows that students who came from an Integrated curricular experience want problems that enable them to apply the mathematics and value gaining a better understanding of the content they study.

Implications for future studies

This study focused on students' past experiences, one which happened to be their experience with various representations of mathematics. Additionally, the confidence of students was examined during interviews, when solving problems which draw on different representations of derivatives. Although only the second task worksheet, which was based on the various representations, was primarily used for this study; more attention could be on the students understanding of the derivative, which is the first task worksheet. Another study could include examining the types of questions asked by the student and teacher during the class period and on tests and homework. Are the students' questions geared more towards clarity? Is the teacher assigning questions of different representations?

Many of the students stated that they did not work on many application problems in their calculus class. However, this study indicates that the teacher did assign some of these problems. This contradiction between what was observed and what the students reported makes one wonder about how much emphasis needs to be given to application problems before students recognize them as part of their experience. A future study could examine what is happening when a teacher assigns the students application problems and goes over the application problems in class, but students don't think they do. Is there some kind of range of emphasis that needs to be given to various types of problems for students to recognize that they have done that kind of strategy? How often would application problems need to be worked on for the students to recognize them?

A combination of a quantitative and qualitative study could be done to get a better sense of the effects of these past curricular experiences. A look at prior tests scores, to get a measure of the students' abilities prior to the different curriculum materials, and survey data that describes the dispositions of the students, could be examined before they entered calculus. Are the students similar or different going into the same class together? What were the similarities and differences before the students begin the different curricular experiences compared to how the students currently are in the calculus experience (where they now come back together)?

Lastly, many of the Traditional students in this study stated that they only wanted to *do* the mathematics and were not concerned with understanding the mathematics. Could it be that they primarily have been asked in all of the other previous mathematics classes to just be able to *do* the mathematics, so that is why much of their conversation focused on that? A document analysis could take place, analyzing the questions from the textbook, the homework problems that were assigned and assessments that were given to see whether the emphasis was more procedural or conceptual. This type of analysis would also help to describe the impact of their past curricular experience, but would require a longitudinal study to examine the phenomena over a long period of time.

What I Learned

First, I learned that organization is a key factor when writing a thesis. Even though I had a short amount of time to collect data, I collected a lot of it. Having data that were organized was helpful when writing up the study. To keep my data organized, right after I conducted interviews, I immediately transcribed the data and looked for emerging themes. I headed each sheet of paper with my research questions and as I found themes, I pasted them under the question it related to. Having a sufficient amount of evidence to support each theme is very important. When I found a theme, I made sure I had evidence. Therefore, when it was time to write up the results from my data, I had all of the themes I was going to discuss as well as the evidence to support my themes. If I would have waited until all the data was collected to find themes, I would have been overwhelmed when it came to finding themes.

Second, I had to think of the thesis as a series of small related tasks. The idea of writing a thesis is an overwhelming feeling. The typical thesis consists of five chapters. The thesis should be written chapter by chapter, but in no specific order. I found it easier to do my literature review section first, after I developed my research questions. I wanted to know what research was out there and whether I felt like my research questions added to the existing studies that were conducted. After I wrote my literature review, I wrote my introduction because I knew then what I had to introduce to the reader before he/she could read my paper. I then wrote my methods and results section around the same time I collected my data. The methods I used were fresh on my mind when I wrote about them, so this made the process easier. Finally, I wrote the discussion section which emphasized why this study is important, among other things. Keep in mind that all of the chapters relate and make sure there is a connection among all of the chapters that make up your thesis.

Third, I learned that studying students' understanding is very difficult. I did not know how to capture a definitive definition of "understanding" and I found this difficult to do and to write. Not only are there are many definitions of understanding, it is a hard thing to measure. I have learned that just because a student answers a problem correctly, it does not mean they truly understand what is going on. How does anyone know if a student truly understands something? Do we have to give the student one test, a set of problems, a pre- and post-test, to test their understanding? How do you know whether those questions are a good measure of their understanding? These and many other questions came to mind when I thought of measuring students' understanding. I wish that I would have been able to include the data I collected on the students' understanding of the definition of derivatives. However, I did not have the time to truly delve into the questions stated above or related questions.

Fourth, I learned a lot about conducting qualitative research. Obtaining students' perspectives is important and conducting qualitative research allows me to do so much more with subtlety than quantitative research would. I learned that for a qualitative study of this type (i.e., master's thesis rather than a dissertation), twelve students are too many students to really understand such a complex phenomena. If I were to do this study again, I would take a smaller sample and really delve deeper into the students' thinking about mathematics and their experiences. Also, I wish I would have had better follow-up interview questions. Now that I look back at my data, I would ask questions that I should have asked the students based on some of their responses. For example, when the students referred to real life problems, I am curious as to what they are referring to as "real life" problems. An Integrated student said in their Integrated classroom, they learned actual mathematics; I would like to see what he is referring to as being "actual mathematics." The students discussed the kinds of problems they do not do in calculus; I would like to hear what kinds of problems they do in calculus.

Fifth and lastly, being a part of a writing group was helpful in the writing process. The members in my writing group gave me encouragement when I needed it. The members were able to reiterate that I was making progress and my writing was improving, although I did not think I was improving. Writing this paper was not like anything I wrote as an undergraduate. As an undergraduate, I remember writing research papers that just required me to present the data. In order to write this thesis, I had to synthesize and analyze not only the data I presented, but the literature that I found about other studies. I had to be explicit about what I did, and more importantly why I chose to do it that way. As I read other studies, I asked questions that were asked to me during my process of writing this thesis. For example, I wanted to know why certain methods were used. Writing this thesis was the hardest, yet most rewarding task that I have conquered throughout my years in education.

Limitations

This study is limited to students from one calculus course in an atypical high school setting[8]. Thus, as with any qualitative analysis, the findings reported here cannot be used to make generalizations to all U.S. students about the impact of their past mathematics experiences and their understanding of various representations of derivatives.

Additionally, the lack of time allotted for the two interviews forced me to ask the students to work on the worksheets outside of the interview. Although the worksheet explicitly asks students to work alone, they could have received help from other students. I had to take the students' words that they did the worksheet individually; this may not have been the case. However, because I had the students explain their reasoning during the interviews, they were accountable for the solutions they wrote.

The interviews, observations, and document analysis took place while the students were being taught introductory concepts related to derivatives. Many of the Integrated students reported that there was not a lot of application of derivatives problems thus far. A later chapter in the textbook does cover applications of derivatives. Some results may have been different if this study had taken place after the students covered this section.

[8] There are high schools in many states that are offering a similar context where students are given a choice between Integrated and Traditional curricula. Some states include North Carolina, Michigan, and Missouri.

APPENDIX A: PARENT CONSENT FORM

Dear parent/guardian:

The Ames School District began offering and Integrated Mathematics sequence in the fall of 2000 as an alternative to the Traditional Algebra sequence.

It is important that we study the effect of these two mathematics options. First, we should know how these two options shape the way students think about mathematics. Second, we should know how these students approach solving a mathematics problem and whether their particular option guided their learning in a certain way. Third, has the use of technology helped or hindered their understanding.

Students with free time during the school day will be selected for an interview that will last 20-25 minutes. To help interpret interviews, I will be observing in the classroom whenever the class focuses on derivative. If your child is selected for an interview, she/he can decide at that time whether or not she/he would like to be interviewed. All data will remain strictly confidential, students' names, will not be used in any reports, and the interview forms will be destroyed at the end of the study. You can withdraw your permission at any time. Regardless of your decision, please sign and have your students return this form to her/his mathematics teacher.

__ I GIVE permission for ______________________________ to participate in this study.
(child's name)

__ I DO NOT GIVE permission for ______________________ to participate in this study.
(child's name)

____________________________ ______________
(parent/guardian name) (date)

Call or email if you have any questions or concerns. Thank you for your time.

Thank you,

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APPENDIX C: TASK WORKSHEETS

Task Worksheet #1

1) What is a derivative?

2) The path of a grapefruit above the ground is given by the values in the table.

t (sec)	0	1	2	3	4	5	6
y (feet)	6	90	142	162	150	106	30

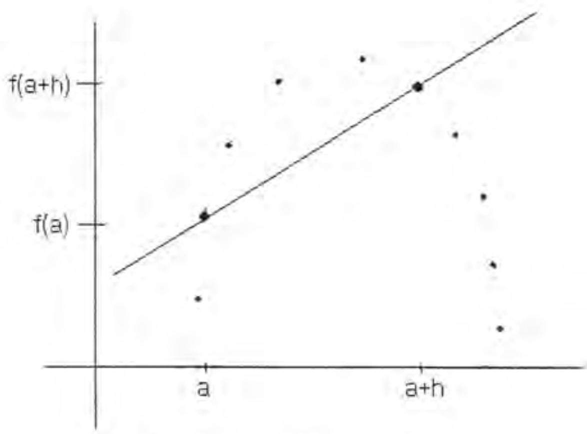
Using the table above, find the average rate of change of:

a. [0,1]

b. [3,2]

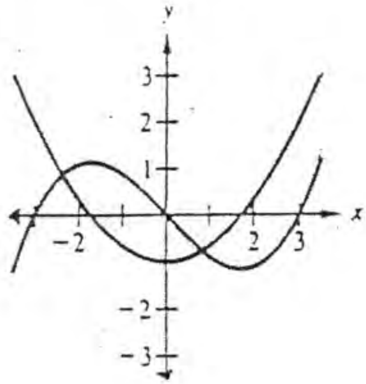
Using the table above estimate the instantaneous velocity at t=2

3) What is the equation below?
Can you explain how the question was derived from the graph below (you can write on the back if you need more space)?



$$\lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

4) Graphs of f and f' appear below. Which is which? How can you tell?



5) Find $f'(3)$ for $f(x) = x^2$ using the definition of the derivative, not the power rule. Is there more than one way to solve this problem? If so, how?

Task Worksheet #2

Please answer the questions below. You may use your graphing calculator.

1. Estimate *s*'(2).

t	0	.5	1	1.5	2	2.5
s	1	2	.5	-20	-43	-74

2. $s(t) = -16t^2 + 10t + 1$. Find $s'(2)$.

3. Graph the equation $-16t^2 - 22t + 270$. Find the derivative graphically

4. Given the position function, $-16t^2 + v_0t + s_0$. A ball is thrown off a 270-foot building with initial velocity of -22 feet per second. What is its initial velocity after 3 seconds?

APPENDIX D: INTERVIEW QUESTIONS

Interview #1:

State your name and sequence taken please.

1) What do people usually mean if they say instantaneous rate of change?
2) What about rate of change?
3) Tell or show me what you knew about derivatives before taking Mr. C's class.
4) What do you know and understand now, that you didn't know or understand before?
5) How confident are you when solving derivatives?
6) How have your past experiences with the courses you've taken help you understand derivatives?
7) Did you receive any help doing the sheet? If so, on what parts did you not understand?
8) Refer to "what is a derivative?" Does anything else come to mind for what a derivative is?
9) How would you explain what is derivative is to someone who is a pre calculus students that hasn't studied it yet?
10) How do you think you are performing in calculus?
11) How has your past courses contributed to you being successful/struggling in calculus?
12) (Referring to number 5 on task worksheet #1), what does your answer mean to the problem?

Interview #2:

State your name and sequence taken please.

1) Out of the four problems on the second task worksheet, which one were you more confident solving? Why were you so confident?
2) What problem did you feel you performed the best on? Why do you think you performed best on those?
3) Can you please explain to me how you got your answers?
4) Did you have any problems completing any of the problems on the worksheet; if so on what numbers and why do you think you had trouble completing that problem?
5) (Ask questions about the first interview that you need some clarification on)

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